Name: __Kevin Braza - KEY________________________________ Date: ______________ IB Math A&A SL

Lesson 1 - The Cartesian Plane, Distance & Midpoint Formula, Equations and Intercepts

1. Consider the points (3,2) and (9,10).

a. Plot the points. Check Desmos

b. Find the distance between the points (3,2) and (9,10).

$$\boldsymbol{d} = \sqrt{(9-3)^{\mathbf{2}} + (10-2)^{\mathbf{2}}}$$

$$\boldsymbol{d} = \sqrt{(6)^{\mathbf{2}} + (8)^{\mathbf{2}}} = \sqrt{\mathbf{36 + 64}} = \sqrt{\mathbf{100}} = \mathbf{10}$$

c. Find the "midpoint" between the points (3,2) and (9,10).

$$(\boldsymbol{x_m}, \boldsymbol{y_m}) = \left(\frac{\mathbf{3+9}}{\mathbf{2}}, \frac{\mathbf{2+10}}{\mathbf{2}}\right) = \left(\frac{\mathbf{12}}{\mathbf{2}}, \frac{\mathbf{12}}{\mathbf{2}}\right) = (\mathbf{6}, \mathbf{6})$$

d. Confirm that the points are located on the equation:

$$y = \frac{4}{3}x - 2$$

$$2 = \frac{4}{3}(3) - 2$$

$$10 = \frac{4}{3}(9) - 2 = (4 * 3) - 2$$

e. Find the y-intercepts of the equation above.

$$y = \frac{4}{3}(0) - 2 = -2$$

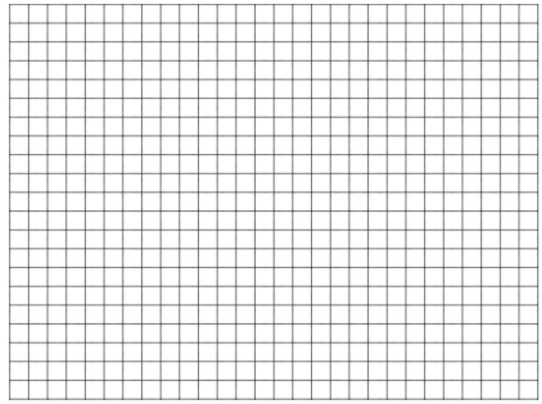
x-intercept: $(0, -2)$

f. Find the x-intercepts of the equation above.

$$0 = \frac{4}{3}x - 2; \quad x = 2 * \frac{3}{4}; \quad x = \frac{3}{2}$$

g. Complete the table below for the equation $y = \frac{4}{3}x - 2$.

x	-3	0	3	6	9
y	-6	-2	2	6	10



The Distance Formula:

$$\boldsymbol{d} = \sqrt{(x_2 - x_1)^{\mathbf{2}} + (y_2 - y_1)^{\mathbf{2}}}$$

(Remember: the distance formula comes from the Pythagorean Theorem, using $(x_2 - x_1)$ and $(y_2 - y_1)$.

The Midpoint Formula:

$$(\boldsymbol{x_m}, \boldsymbol{y_m}) = \left(\frac{\boldsymbol{x_1 + x_2}}{\mathbf{2}}, \frac{\boldsymbol{y_1 + y_2}}{\mathbf{2}}\right)$$

(The midpoint is just average of the x values and the average of the y values.)

Finding Intercepts:

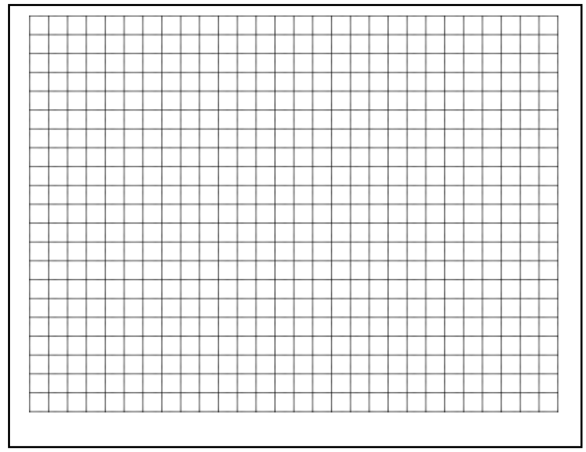
y-intercepts: The value touching the y-axis.

Set x = 0 and solve for y.

x-intercepts: The value(s) touching the x-axis.

Set y = 0 and solve for x.

2. Consider the points (-2, -3) and (2,5).
   a. Plot the points. Check Desmos.
   b. Find the distance between the points.



$$d = \sqrt{(2+2)^2 + (5+3)^2}$$

$$d = \sqrt{(4)^2 + (8)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$$

   c. Find the midpoint between the points.

$$(x_m, y_m) = \left(\frac{-2+2}{2}, \frac{-3+5}{2}\right) = \left(\frac{0}{2}, \frac{2}{2}\right) = (0, 1)$$

   d. Confirm that the two points lie on the equation:
      $y = x^2 + 2x - 3$
      $-3 = (-2)^2 + 2(-2) - 3 = 4 - 4 - 3 = -3$ $\quad$ $5 = (2)^2 + 2(2) - 3 = 4 + 4 - 3 = 5$
   e. Complete the table below and graph the equation: $y = x^2 + 2x - 3$

x	-4	-3	-2	-1	0	1	2
y	5	0	-3	-4	-3	0	5

Practice: For problems #3 and #4, find the y-intercepts and x-intercepts of the following equations:

3. $y = x^3 - 4x$

$y = (0)^3 - 4(0) = 0$ $\quad$ $0 = x(x^2 - 4) = x(x-2)(x+2)$

y-intercept: (0,0) $\quad$ x-intercepts: (0,0); (2,0); (-2,0)

4. $y^2 + 2x = 16$

$y^2 + 2(0) = 16$ $\quad$ $y^2 = 16$ $\quad$ $y = \pm 4$ $\quad$ y-intercepts: (-4,0); (4,0)

$(0)^2 + 2x = 16$ $\quad$ $2x = 16$ $\quad$ $x = 8$ $\quad$ x-intercepts: (8,0)

5. Show that these points form the vertices of an isosceles triangle. $(1, -3); (3,2); (-2,4)$.

$$d_{2nd-1st} = \sqrt{(3-1)^2 + (2+3)^2} = \sqrt{(2)^2 + (5)^2} = \sqrt{4+25} = \sqrt{29}$$

$$d_{3rd-2nd} = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{(-5)^2 + (2)^2} = \sqrt{25+4} = \sqrt{29}$$

$$d_{3rd-1st} = \sqrt{(-2-1)^2 + (4+3)^2} = \sqrt{(-3)^2 + (7)^2} = \sqrt{9+49} = \sqrt{58}$$

Only 2 sides are of equal length = isosceles triangle.

6. Consider the coordinates $A(-1,1), B(3,6), C(6,2), and\ D(2,-3)$. Work with a partner to find the lengths of $\overline{AB}, \overline{BC}, \overline{CD}, \overline{AD}$. What figure is represented by the quadrilateral formed by these four lengths?

$\overline{\boldsymbol{AB}} = \sqrt{(3+1)^{\mathbf{2}} + (6-1)^{\mathbf{2}}} = \sqrt{(4)^{\mathbf{2}} + (5)^{\mathbf{2}}} = \sqrt{\mathbf{16+25}} = \sqrt{\mathbf{31}}$

$\overline{\boldsymbol{BC}} = \sqrt{(6-3)^{\mathbf{2}} + (2-6)^{\mathbf{2}}} = \sqrt{(3)^{\mathbf{2}} + (-4)^{\mathbf{2}}} = \sqrt{\mathbf{9+16}} = \sqrt{\mathbf{25}} = \mathbf{5}$

$\overline{\boldsymbol{CD}} = \sqrt{(2-6)^{\mathbf{2}} + (-3-2)^{\mathbf{2}}} = \sqrt{(-4)^{\mathbf{2}} + (-5)^{\mathbf{2}}} = \sqrt{\mathbf{16+25}} = \sqrt{\mathbf{31}}$

$\overline{\boldsymbol{AD}} = \sqrt{(2+1)^{\mathbf{2}} + (-3-1)^{\mathbf{2}}} = \sqrt{(3)^{\mathbf{2}} + (-4)^{\mathbf{2}}} = \sqrt{\mathbf{9+16}} = \sqrt{\mathbf{25}} = \mathbf{5}$

Opposite sides are the same length but the angles are not 90 degrees = this is a parallelogram.