Name: ___Kevin Braza – KEY______________________________ Date: __________________ IB Math A&A SL

Lesson 1.8 – Inverse Functions

What is an inverse function?

$$x \to f(x) \to y$$

$$x \leftarrow f^{-1}(x) \leftarrow y$$

x	$f(x)$	$g(x)$
-4	3	10
-3	-4	-1
-2	0	4
-1	1	3
0	10	2
1	5	1
2	2	0
3	8	-5
4	-2	-2

1. Find x such that $f(x) = 5$

$f(x) = 5$ when $x = 1$

2. Evaluate the following:

a. $f^{-1}(5) + g^{-1}(3)$

$f^{-1}(5) + g^{-1}(3) = 1 + (-1) = 0$

b. $f^{-1} \circ g^{-1}(-5)$

$f(3) = 8$

c. $f^{-1} \circ f^{-1}(8)$

$f^{-1}(3) = -4$

d. $g^{-1} \circ g^{-1}(3) + g(3)$

$g^{-1}(-1) - 5 = -3 - 5 = -8$

e. $f^{-1} \circ g^{-1}(4)$

$f^{-1}(-2) = 4$

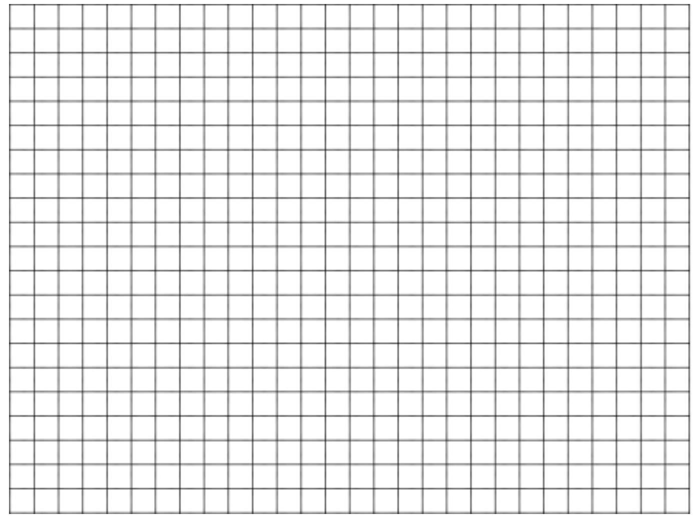
f. $f \circ f^{-1}(3)$

$f(-4) = 3$ (Composition of a function with its inverse will cancel and yield only the argument).

I. Inverse Functions Graphically

1. Use your calculator to sketch a graph of $f(x) = \frac{1}{4}x^3$.

Check Desmos



2. On the same graph, use your calculator (or any other means) to help you sketch a graph of $g(x) = \sqrt[3]{4x}$.

Check Desmos

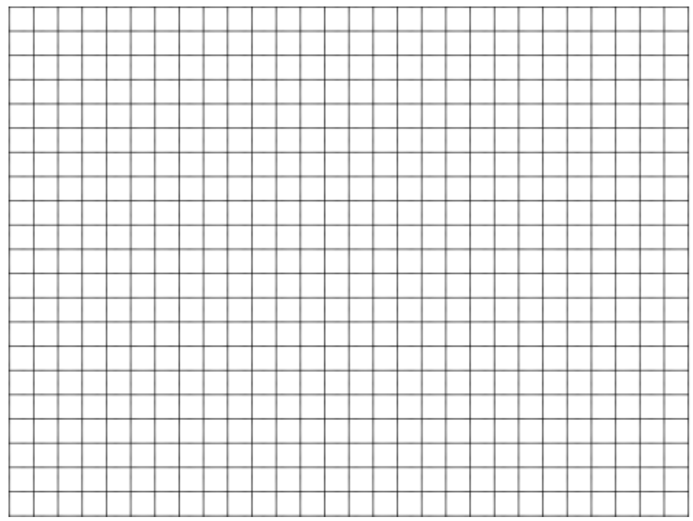
3. Visually, what is the relationship between the two functions?

Reflection across $y = x$

Finding inverse functions graphically: $f(x) \rightarrow (x, y)$	$f^{-1}(x) \rightarrow (y, x)$

4. Find the "inverse function" of $f(x) = x^2$.

Check Desmos: $f^{-1}(x) = \pm\sqrt{x}$



5. Graph $f(x)$ and its "inverse" $f^{-1}(x)$ to the right. What is wrong with this function?

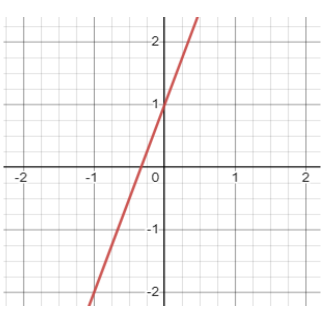
Not a function

6. What is a **one-to-one function**?

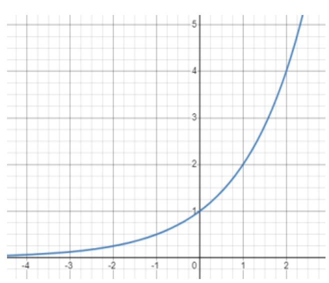
If no two elements in the domain of f correspond to the same element in the range of f.

7. Graph the inverse of each function listed below.

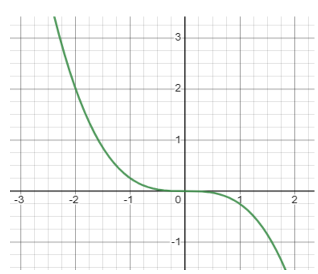
$f(x) = 3x + 1$



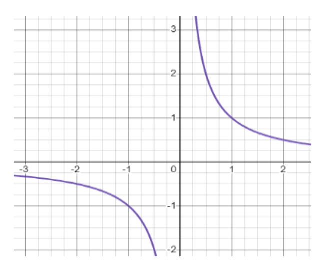
$f(x) = 2^x$



$f(x) = -\frac{1}{4}x^3$



$f(x) = \frac{1}{x}$



Check Desmos for each.

II. Inverse Function Algebraically

Finding inverse functions algebraically:

Switch x and y and solve for y.

8. Find the inverse function of $f(x) = 3x + 1$.

$$x = 3y + 1$$

$$y = \frac{1}{3}(x - 1)$$

9. $g(x) = \sqrt{x + 2}$

Find $g^{-1}(x)$, then show that $g \circ g^{-1}(x) = x$

$x = \sqrt{y + 2}$ $\quad$ $g \circ g^{-1}(x) = \sqrt{x^2 - 2 + 2}$

$x^2 = y + 2$ $\quad$ $g \circ g^{-1}(x) = \sqrt{x^2}$

$y = x^2 - 2$ $\quad$ $g \circ g^{-1}(x) = x$

10. $h(x) = \frac{x-3}{x+2}$

Find $h^{-1}(x)$, then show $h \circ h^{-1}(x) = x$

$x = \frac{y-3}{y+2}$ $\quad$ $h \circ h^{-1}(x) = \frac{\left(\frac{-2x-3}{x-1}\right)-3}{\left(\frac{-2x-3}{x-1}\right)+2}$ $\quad$ $h \circ h^{-1}(x) = x$

$xy + 2x = y - 3$ $\quad$ $h \circ h^{-1}(x) = \frac{\left(\frac{-2x-3-3(x-1)}{x-1}\right)}{\left(\frac{-2x-3+2(x-1)}{x-1}\right)}$

$xy - y = -2x - 3$ $\quad$ $h \circ h^{-1}(x) = \frac{-2x-3-3(x-1)}{-2x-3+2(x-1)}$

$y(x - 1) = -2x - 3$ $\quad$ $h \circ h^{-1}(x) = \frac{-2x-3-3x+3}{-2x-3+2x-2}$

$y = \frac{-2x-3}{x-1}$ $\quad$ $h \circ h^{-1}(x) = \frac{-5x}{-5}$

11. Let $m(x) = \frac{x-4}{2}$ and $f(x) = 3x + 1$. Evaluate the following.

$$f^{-1}(x) = \frac{1}{3}(x - 1);\ x = \frac{y - 4}{2} \rightarrow y = 2x + 4 \rightarrow\ m^{-1}(x) = 2x + 4$$

a. $f^{-1} \circ m^{-1}(x)$

$$\frac{1}{3}\big((2x + 4) - 1\big) = \frac{1}{3}(2x + 3) = \frac{2}{3}x + 1$$

b. $(f \circ m)^{-1}(x)$

$$f \circ m(x) \rightarrow y = 3\left(\frac{x - 4}{2}\right) + 1$$

$$(f \circ m)^{-1}(x) \rightarrow x = 3\left(\frac{y - 4}{2}\right) + 1 \rightarrow x - 1 = \frac{3}{2}(y - 4) \rightarrow y = \frac{2}{3}(x - 1) + 4$$

c. $m^{-1} \circ f^{-1}(x)$

$$\frac{(3x + 1) - 4}{2} = \frac{3x - 3}{2}$$