Name: ______Kevin Braza - KEY__________________________ Date: _________________ IB Math A&A SL

Lesson 2.1 – Quadratic Functions

Analyzing Quadratic Functions:

Standard Form of a Quadratic Function:

$y = ax^2 + bx + c$

Vertex Form of a Quadratic Function:

$y = a(x - h)^2 + k$

(The "vertex form" is the form a quadratic equation takes after completing the square.)

Be able to find the following features of a parabola: Vertex, axis of symmetry, x and y-intercepts.

I. Graphing Quadratic Functions

For each function below, complete the square, identify the vertex, axis of symmetry & x-intercept(s). Sketch a graph for the first two.

1. $f(x) = x^2 + 6x + 5$

$x^2 + 6x\ [+9 - 9] + 5$

$(x + 3)^2 - 4$

Vertex: $(-3, -4)$

Axis of Symmetry: $x = -3$

$y = 0 + 0 + 5 = 5$

y-intercept: $(0, -5)$

$0 = x^2 + 6x + 5$

$0 = (x + 5)(x + 1)$

$x = -5, x = -1$

x-intercepts:
$(-5,0); (-1,0)$

Check graph on Desmos.

2. $f(x) = 2x^2 - 12x + 10$

$2(x^2 - 6x) + 10$

$2(x^2 - 6x\ [+9 - 9]) + 10$

$2((x-3)^2 - 9) + 10$

$2(x-3)^2 - 18 + 10$

$2(x-3)^2 - 8$

Vertex: $(3, -8)$

Axis of Symmetry: $x = 3$

$y = 0 + 0 + 10 = 10$

y-intercept: (0,10)

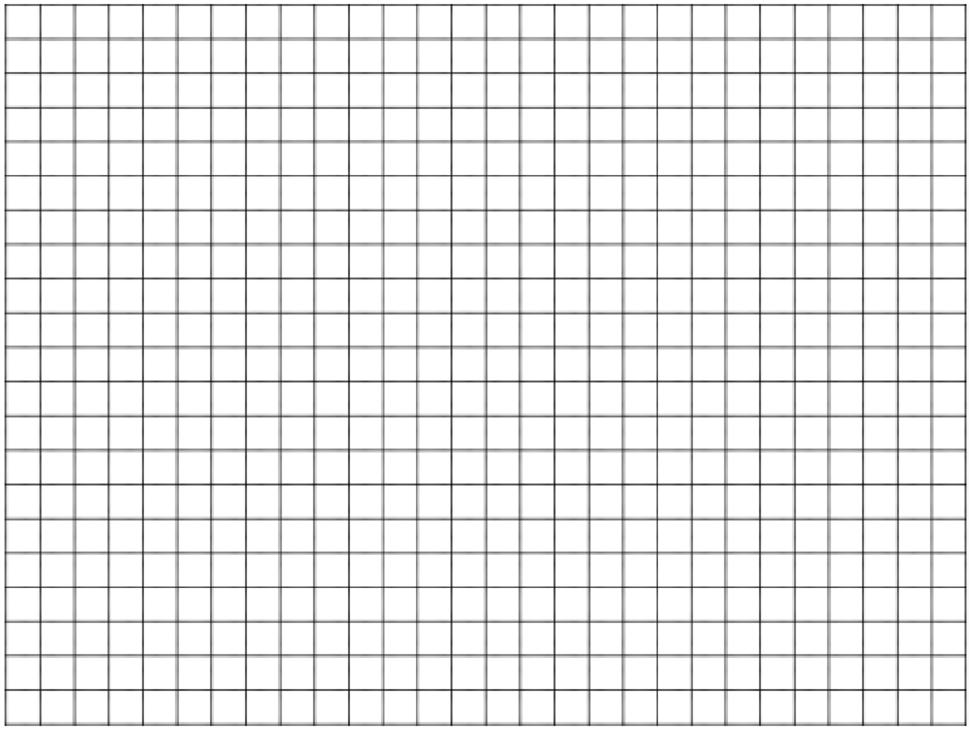
$0 = 2x^2 - 12x + 10$

$0 = 2(x^2 - 6x + 5)$

$0 = (x-5)(x-1)$

$x = 5, x = 1$

x-intercepts: (5,0); (1,0)



Check Graph on Desmos.

3. $f(x) = -4x^2 + 24x - 41$

$-4(x^2 - 6x) - 41$

$-4(x^2 - 6x\ [+9 - 9]) - 41$

$-4((x-3)^2 - 9) - 41$

$-4(x-3)^2 + 36 - 41$

$-4(x-3)^2 - 5$

Vertex: $(3, -5)$

Axis of Symmetry: $x = 3$

$y = 0 + 0 - 41 = -41$

y-intercept: $(0, -41)$

$0 = -4(x-3)^2 - 5$

$5 = -4(x-3)^2$

$-\frac{5}{4} = (x-3)^2$

x-intercepts: None

4. $f(x) = 2x^2 - x - 1$

$2(x^2 - \frac{1}{2}x) - 1$

$2\left(x^2 - \frac{1}{2}x\left[+\left(\frac{-1}{4}\right)^2 - \left(\frac{-1}{4}\right)^2\right]\right) - 1$

$2(\left(x - \frac{1}{4}\right)^2 - \left(\frac{-1}{4}\right)^2) - 1$

$2(\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}) - 1$

$2\left(x - \frac{1}{4}\right)^2 - \frac{1}{8} - 1$

$2\left(x - \frac{1}{4}\right)^2 - \frac{9}{8}$

Vertex: $(\frac{1}{4}, -\frac{9}{8})$

Axis of Symmetry: $x = \frac{1}{4}$

$y = 0 - 0 - 1 = -1$

y-intercept: $(0, -1)$

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x-1) + 1(x-1) = 0$

$(2x+1)(x-1) = 0$

$x = -\frac{1}{2}, x = 1$

x-intercepts: $\left(-\frac{1}{2}, 0\right)$; (1,0)

II. Finding the Leading Coefficient ‘a’

1. Find the quadratic equation whose vertex is (4, -1) and passes through the point (2, -5).

$y = a(x-h)^2 + k$

$y = a(x-4)^2 - 1$

$-5 = a(2-4)^2 - 1$

$-4 = a(4)$

$a = -1$

$y = -(x-4)^2 - 1$

2. Find the equation of a parabola that has the vertex (2, 3) and passes through the point (0, 2).

$y = a(x-h)^2 + k$

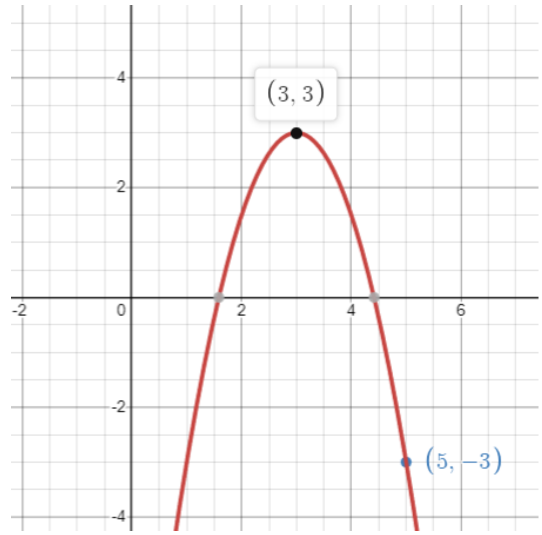
$y = a(x-2)^2 + 3$

$2 = a(0-2)^2 + 3$

$-1 = a(4)$

$a = -\frac{1}{4}$

$y = -\frac{1}{4}(x-2)^2 + 3$



3. Write the equation of the function for the graph to the right.

$y = a(x-h)^2 + k$

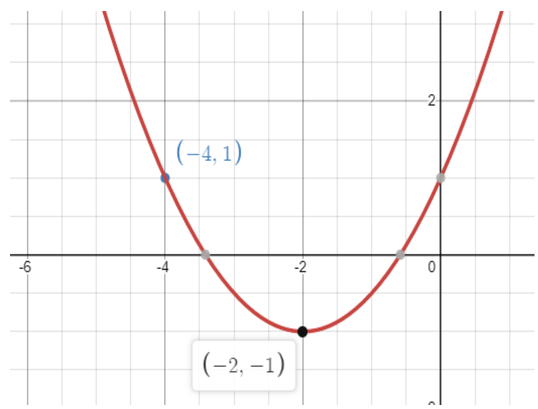
$y = a(x-3)^2 + 3$

$-3 = a(5-3)^2 + 3$

$-6 = a(4)$

$a = -\frac{-3}{2}$

$y = -\frac{-3}{2}(x-3)^2 + 3$



4. Write the equation of the function for the graph to the right.

$y = a(x-h)^2 + k$

$y = a(x+2)^2 - 1$

$1 = a(-4+2)^2 - 1$

$2 = a(4)$

$a = \frac{1}{2}$

$y = \frac{1}{2}(x+2)^2 - 1$

5. Rewrite a general quadratic equation $ax^2 + bx + c = 0$ in vertex form. What do you notice about the axis of symmetry for a general quadratic equation?

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x\ [+\left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2] + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2} = 0$

$(x + \frac{b}{2a})^2 + \frac{4ac-b^2}{4a^2} = 0$

Axis of Symmetry: $x = \frac{b}{2a}$