Name: ___Kevin Braza - KEY_____ Date: _____ IB Math A&A SL

Lesson 2.2 – Modeling & Optimization with Quadratic Functions

1. ***Physics*** - The path of a diver is given by
$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

Where y is the height (in feet) and x is the horizontal distance from the end of the diving board (in feet). What is the maximum height of the diver?

Axis of Symmetry: $x = \frac{-b}{2a} = \frac{-\frac{24}{9}}{2(\frac{-4}{9})} = \frac{-\frac{24}{9}}{\frac{-8}{9}} = \frac{-24}{-8} = 3$

$y = -\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12$

$y = -\frac{4}{9}(9) + \frac{24}{9}(3) + 12$

$y = -4 + \frac{24}{3} + 12$

$y = -4 + 8 + 12$

$y = 16$

Vertex: (3,16)

2. ***Economics*** – A textile manufacturer has daily production costs of $C = 100{,}000 - 110x + 0.045x^2$ where C is the total cost (in dollars) and x is the number of units produces. How many units should be produced each day to yield a minimum cost?

Axis of Symmetry: $x = \frac{-b}{2a} = \frac{-110}{2(0.045)} = \frac{-110}{0.09} = 1222\ units$

3. ***Finance*** – The profit P (in hundreds of dollars) that a company makes depends on the amount x (in hundreds of dollars) the company spends on advertising according to the model:
$P = 230 + 20x - 0.5x^2$

   a. What expenditure for advertising will yield a maximum profit?

Axis of Symmetry: $x = \frac{-b}{2a} = \frac{-20}{2(-0.5)} = \frac{-20}{1} = 20\ (hundreds\ of\ dollars)$

   b. What is the maximum profit?

$P = 230 + 20(20) - 0.5(20)^2 = 430$

4. ***Sports*** – A football kicker kicks a football a total distance of 150 feet. The ball reached a maximum height of 80 feet. Write an equation that models the height y to the distance x. (Assume that the motion of a projectile is quadratic).

Vertex $(75, 80)$

Other Points: $(0,0); (150,0)$

$-80 = a(-75)^2$

$a = \frac{-80}{5625} = \frac{-16}{1125}$

$y = a(x-h)^2 + k$

$y = a(x-75)^2 + 80$

$0 = a(0-75)^2 + 80$

$y = -\frac{-16}{1125}(x-75)^2 + 80$

5. ***Forensics*** – CSI detectives drop mannequins from a building to help them decide if a victim feel off a building or was pushed off of a building. A mannequin was lightly pushed out of an office 450 feet above ground level and landed 15 feet from base of the building. (Assume that the motion of a projectile is quadratic). Write an equation that models height *y* to the distance the mannequin travels *x*.

Vertex $(0, 450)$

Other Points: $(15,0)$

$y = a(x - 0)^2 + 450$

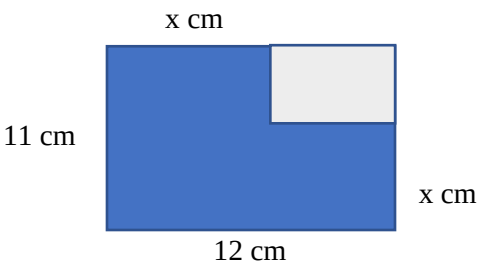
$0 = a(15)^2 + 450$

$-450 = a(225)$

$a = \frac{-450}{225} = -2$

$y = -2x^2 + 450$

6. ***Geometry*** **-** The area of the shaded part is 112 cm^2. Solve for the value of x.



$(12 - x)(11 - x) = 132 - 112$

$132 - 11x - 12x + x^2 = 132 - 112$

$112 - 23x + x^2 = 0$

$x^2 - 23x + 112 = 0$

$(x - 7)(x - 16) = 0$

$x = 7, \sout{x = 16}$