Name: __Kevin Braza – KEY ________________________________ Date: _____________ IB Math A&A SL

Lesson 3.10 - Logarithmic & Exponential Models II

1. ***Turkey Cooking Times!*** - A turkey is considered "fully-cooked" when the internal temperature of the turkey reaches 165°F. The starting temperature of a turkey is measured at 40°F and then it is placed into an oven at 325°F. The internal temperature of the turkey can be modeled by the equation.

$$F(t) = -285e^{kt} + 325$$

Where F(t) describes the internal temperature of the turkey and t describes the time in minutes. Mr. Braza notices that his turkey's internal temperature reaches 54.6°F after 30 minutes.

a. Determine the internal temperature of the turkey after 3 hours.

$54.6 = -285e^{30k} + 325$

$-270.4 = -285e^{30k}$

$\ln\frac{270.4}{285} = 30k$

$k = -0.001753$

$F(180) = -285e^{180(-0.001753)} + 325$

$F(180) = 117.122°\text{F}$

b. Determine the time it takes for the turkey to be considered "fully cooked"

$165°\text{F} = -285e^{(-0.001753t)} + 325$

$-160 = -285e^{(-0.001753t)}$

$\frac{\ln\frac{32}{57}}{-0.001753} = t = 329.33 \text{ min} = 5\frac{1}{2}\ hr$

c. Graph the function from $0 \le t \le 2500$ using the k value you solved for. What happens to the internal temperature of the turkey after a long, long time?

As $t \to \infty, F(t) \to$ flattens out at $5.3E + 3.325$

2. On a college campus of 7500 students, one student returns from vacation with a contagious and long-lasting virus. The spread of the virus is modeled by:

$$V(t) = \frac{7500}{1 + 7499e^{-0.9t}},\ t \ge 0$$

Where $V(t)$ is the number of students afflicted by the virus after t days.

a. How many students will be infected after 4 days?

$V(4) = \frac{7500}{1 + 7499e^{-0.9(4)}} = 36\ students$

b. The college will cancel classes when 30% or more of the students are infected. After how many days will the college cancel classes?

$2250 = \frac{7500}{1+7499e^{-0.9t}}$

$2250 + 16872750e^{-0.9t} = 7500$

$e^{-0.9t} = \frac{7}{22497}$

$t = 8.97 \approx 9\ days$

c. Graph the function for $0 \le t \le 30$. If classes were not cancelled, explain the end behavior of the spread of the virus on the college campus.

The virus would spread to the entire population and stop at 7500 infected students.

3. The management at plastics factory has found that the maximum number of units a worker can produce in a day is 30. The learning curve for the number *N* of units produced per day after a new employee has worked *t* days is modeled by $N = 30(1 - e^{kt})$. After being on the job 20 days, a worker can produce 19 units in a day,

a. Find the learning curve for this employee (find the value of 'k').

$19 = 30\left(1 - e^{k(20)}\right)$

$e^{20k} = 1 - \frac{19}{30}$

$\frac{19}{30} = 1 - e^{20k}$

$k = \frac{\ln(1 - \frac{19}{30})}{20} = -0.050165$

b. How many days should pass before this employee is producing 25 units a day?

$N = 30\left(1 - e^{-0.050165(25\,yr)}\right) = 21.44\ days$