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Lesson 3.3 - Continuous Compounding & the value 'e'

I. Warm-Up – Martha Stewart deposits $1 in an account at a very generous bank that pays her 100% interest. Assuming no other deposits and withdrawals, what will her balance be in one year if the interest is compounded:

(a). Quarterly

$A = 1(1 + \frac{1.00}{1})^{1*1yr}$

2

(b). Monthly

$A = 1(1 + \frac{1.00}{12})^{12*1yr}$

2.61

(c). Daily

$A = 1(1 + \frac{1.00}{365})^{365*1yr}$

2.71

d. What do you notice? Can you write a function that gives you the balance after *n* compoundings in 1 year?

$A = 1(1 + \frac{1.00}{n})^{n*1yr}$ Graph this on Desmos and watch what happens as n gets larger and larger.

e. **Key Idea** – Will Martha's ending balance ever exceed $3? No, 2.71 is the limit.

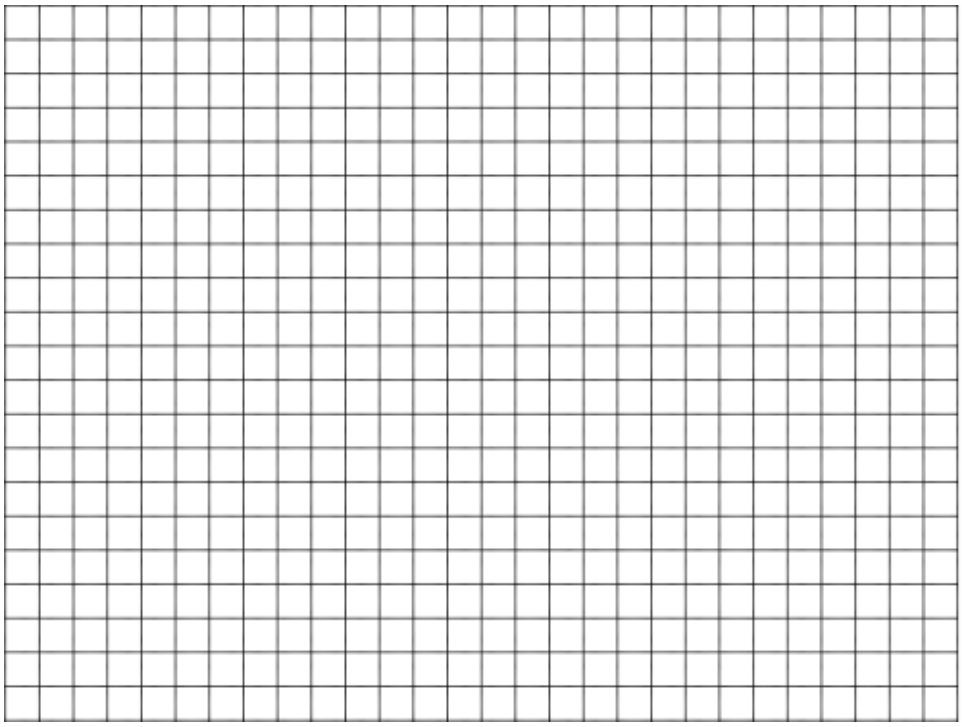
II. The Value 'e' = **2.71828** = $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$

2. Sketch a graph of $y = e^x$.

Check Desmos.

3. One the same graph, sketch a graph of $y = 3e^x + 1$.

Check Desmos.



4. Evaluate the following functions at the indicated values: (Using your calculator)

(a). $y = e^x$ at $x = 3.2$

24.533

(b). $y = 1.5e^{\frac{x}{2}}$ at $x = 24$

244132.187

(c). $y = 250e^{0.05x}$ at $x = 20$

679.570

III. Continuous Compound Interest

Normal Compound Interest Formula	Continuous Compounding Interest Formula
$A = P(1 + \frac{r}{n})^{nt}$	Let $u = \frac{n}{r}$, $A = Pe^{rt}$

5. How much will a $100 deposit earning 6% interest, compounded monthly, yield in 5 years?

$A = P(1 + \frac{r}{n})^{nt} = \$100(1 + \frac{0.06}{12})^{(12*5yr)} = \$134.89$

6. How much will a $100 deposit earning 6% interest, compounded continuously, yield in 5 years?

$A = Pe^{rt} = \$100(e^{0.06*5yr}) = \$134.99$

7. Boruto's dad creates a trust fund for Boruto when he is born and deposits 10,000 両 (ryo). The trust fund pays 9% interest compounded continuously. Determine the balance of this account when Boruto goes to college at the age of 18.

$A = Pe^{rt} = 10{,}000$ 両 $(e^{0.09*18yr}) = 50530$ 両

8. The population P (in mllions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0039t}$, where t represents the year and $t = 6$ corresponding to 1996.

a. According to the model, is the population of Russia increasing or decreasing? Explain.

This is a graph of exponential decay. The population of Russia is decreasing.

b. Find the population of Russia in the year 1998.

$P(1998) = 152.26e^{-0.0039(1998)} = 147.58$ (million)

c. Find the population of Russia in the year 2001.

$P(2001) = 152.26e^{-0.0039(2001)} = 145.87$ (million)

IV. Looking Ahead – Suppose I had an initial deposit of $10,000. When does my balance reach $1,000,000 at an annual compounding of 5% interest rate?

Graph it on Desmos. Continuous → 276 years

Annual → 283 years