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Lesson 3.4 - Introduction to Logarithms

I. What are Logarithms? $\boldsymbol{y = a^x \Leftrightarrow \log_a y = x}$

For example:

We use logarithms to solve for variable exponents.

$\log_2 8 = 3;\ \log_{10} 100000 = 5;$

$\boldsymbol{f(x) = 2^x, f^{-1}(x) = \log_2 x}$

$\log_2 \sqrt{2} = \frac{1}{2}$

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1. Evaluate the following without a calculator.
   a. $\log_2(16)$ 4
   b. $\log_{10}(1)$ 0
   c. $\log_3(27)$ 3
   d. $\log_8(8)$ 1
   e. $\log_4(2)$ $\frac{1}{2}$
   f. $\log_{10}\left(\frac{1}{1000}\right)$ -3

2. Use a calculator to find the following.
   a. $\log_{10}(10) = 1$
   b. $3 \cdot \log_{10}(2) = 0.90309$
   c. $\log_{10}(-4) = undefined$

II. Properties of Logs:

a. $\log_a(1) = 0$

b. $\log_a(a) = 1$

c. $\log_a(a)^x = x$

What does the "common logarithm" mean? $\log x = \log_{10} x$

What does the "natural logarithm" mean? $\log_e x = \ln x$

III. Rewriting from logarithmic form to exponential form

3. Solve each logarithmic equation:

   a. $\log_{16}(x) = \frac{3}{4}$

$16^{3/4} = x;\ (16^{1/4})^3 = x;\ \ 2^3 = x$

$x = 2$

   b. $\ln(x) = 2$

$e^2 = x$

$x = 7.3891$

   c. $6 \cdot \log_3(x+1) = 13$

$6\log_3 x = 12$

$\log_3 x = 2$

$3^2 = x$ $\quad$ $x = 9$

4. Solve each exponential equation:

a. $5^x = 30$ b. $e^x = 7$ c. $5 \cdot 7^x = 3$

$\log_5 30 = x$

$x = 2.11328$

$\ln(7) = x$

$x = 1.94591$

$7^x = \frac{3}{5}$

$\log_7 \frac{3}{5} = x = -0.262512$

IV. Practice

5. $\log_4(x+1) = -\frac{1}{2}$

$4^{-\frac{1}{2}} = x + 1$

$\frac{1}{2} = x + 1$

$1 = 2x + 2$

$2x = -1$

$x = -\frac{1}{2}$

6. $2(6^{5x}) = 120$

$6^{5x} = 60$

$\log_6 60 = 5x$

$x = \frac{1}{5}\log_6 60$

$x = 0.45702$

7. $\ln(x) - 1 = 7$

$\ln(x) = 8$

$e^8 = x = 2980.96$

8. $4e^x = 91$

$e^x = \frac{91}{4}$

$\ln\left(\frac{91}{4}\right) = x$

$x = 3.12457$