Name: ____Kevin Braza – KEY _____ Date: _____ IB Math A&A SL

Lesson 3.6 - Properties of Logarithms

I. Properties of Logs

Let *a* be a positive number such that $a \neq 1$, and let *n* be a real number. If *u* and *v* are positive real numbers, the following properties are true:

Examples:

i. $\log_a(uv) = \log_a(u) + \log_a(v)$

$a^u a^v = a^{u+v}$

$\log(3x) = \log(3) + \log(x)$

$\ln 2 + \ln x + \ln y = \ln(2xy)$

ii. $\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$

$\frac{a^u}{a^v}$

$\log\left(\frac{3}{x}\right) = \log 3 - \log x$

$\log\left(\frac{3xy}{z}\right) = \log(3xy) - \log(z)$

iii. $\log_a(u^n) = n \cdot \log_a(u)$

$(a^u)^n = a^{u*n}$

$\log(x^3) = 3\log x$

$\log 3y^4 = \log 3 + 4\log y$

iv. $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

$y = \log_a(x) \Leftrightarrow a^y = x$

$\log_b a^y = \log_b x$

$y \cdot \log_b a = \log_b x$

$y = \frac{\log_b x}{\log_b a}$

$\log\frac{3x^2}{yz^3} = \log 3 + 2\log x - \log y - 3\log z$

$\log\frac{\sqrt{x+1}}{y^2 z} = \frac{1}{2}\ln(x+1) - 2\ln y - \ln z$

Use the properties of logs to expand the following:

1. $\log(3x^4y^2)$

$\log 3 + 4\log x + 2\log y$

2. $\log\left(\frac{\sqrt{3x-5}}{7x^3}\right)$

$\frac{1}{2}\log(3x-5) - 3\log 7x$

3. $\log\left[\left(\frac{4x^3}{y}\right)^2\right]$

$2\log(\frac{4x^3}{y})$

$2(\log 4 + 3\log x - \log y)$

$2\log 4 + 6\log x - 2\log y$

Use the properties of logs to condense the following:

4. $2\log(x+2)-\frac{1}{3}[\log(x)+\log(y)]$

$2\log(x+2)-\frac{1}{3}(\log xy)$

$\log(x+2)^2-\log\sqrt[3]{xy}$

$\log\frac{(x+2)^2}{\sqrt[3]{xy}}$

5. $\log(x)-3\log(x+1)$

$\log x-\log(x+1)^3$

$\log(\frac{x}{(x+1)^3})$

6. $\log_5(75)-\log_5(3)$

$\log_5\frac{75}{3}$

$\log_5 25$

2

If $\log(2)\approx 0.301$ and $\log(7)\approx 0.845$ find the following <u>without</u> a calculator (show work).

7. $\log(2^3)$

$3(0.301)$

0.903

8. $\log(14)$

$\log 7+\log 2$

$0.845+0.301$

1.146

9. $\log(20)$

$\log 2+\log 10$

$0.301+1$

1.301

10. $\log(7000)$

$\log 7+\log 1000$

$0.845+3$

3.845

11. $\log\left(\frac{1}{7}\right)$

$\log 7^{-1}$

$-\log 7$

-0.845

12. $\log(5)$

$\log 10-\log 2$

$1-0.301$

0.699

13. Evaluate without a calculator: $\frac{\log_3(2)}{\log_3(8)}=\log_8 2=\frac{1}{3}$

14. Show that $\log(3)\cdot\ln(10)=\ln(3)$

$\frac{\ln 3}{\ln 10}\cdot\ln 10=\ln 3$

$\ln 3=\ln 3$

15. Show that $\log_{\frac{1}{4}}(x)=-\log_4(x)$

$\frac{\log_4 x}{\log_4\frac{1}{4}}=-\log_4 x$

$\frac{\log_4 x}{-1}=-\log_4 x$

$\log_4 x=\log_4 x$