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Lesson 3.7 - Solving Exponential Equations

I. Solving Exponential Equations

Solve for x. Discern situations when you need or don't need a calculator.

1. $9^x = 27^{2-x}$

$3^{2x} = 3^{6-3x}$ $\quad 5x = 6$ $\quad x = \frac{6}{5}$

$2x = 6 - 3x$

2. $e^{2x} - 3e^x + 2 = 0$

$u^2 - 3u + 2 = 0$

$(u-2)(u-1) = 0$

$u = 2, u = 1$

$e^x = 2, e^x = 1$

$x = \ln 2, x = 0$

3. $3^{6-x} + 13 = 40$

$3^{6-x} = 27$ $\quad 6 - x = 3$

$3^{6-x} = 3^3$ $\quad x = 3$

4. $\frac{8}{5^{x+2}} = \frac{1}{2^{x-2}}$

$5^{x+2} = 8 \cdot 2^{x-2}$

$5^{x+2} = 2^3 \cdot 2^{x-2}$

$5^{x+2} = 2^{x+1}$

$\log_2 5^{x+2} = x + 1$

$(x+2)\log_2 5 = x + 1$

$x \log_2 5 + 2\log_2 5 = x + 1$

$x \log_2 5 - x = 1 - 2\log_2 5$

$x(\log_2 5 - 1) = 1 - 2\log_2 5$

$x = \frac{1 - 2\log_2 5}{\log_2 5 - 1}$

$x = -2.76$

II. Practice

1. $32^x - 1 = -\frac{7}{8}$

$2^{5x} = -\frac{7}{8} + \frac{8}{8} = -\frac{1}{8}$ $\quad 2^{5x} = 2^{-3}$ $\quad 5x = -3$ $\quad x = -\frac{3}{5}$

2. $8^{2x-1} = 16^{1-x}$

$2^{3(2x-1)} = 2^{4(1-x)}$ $\quad 2^{6x-3} = 2^{4-4x}$ $\quad 6x - 3 = 4 - 4x$ $\quad 10x = 7$ $\quad x = \frac{7}{10}$

3. $\frac{400}{1+e^{-x}} = 350$

$400 = 350 + 350e^{-x}$ $\quad 50 = \frac{350}{e^x}$ $\quad e^x = \frac{350}{50} = 7$ $\quad x = \ln 7 = 1.946$

4. $4^{x+3} = 7^x$

$x + 3 = \mathrm{x} \cdot \log_4 7$ $\quad x - x \cdot \log_4 7 = -3$ $\quad x(1 - \log_4 7) = -3$ $\quad x = -\frac{3}{1 - \log_4 7}$

5. $e^{2x} - e^x - 6 = 0$

$u^2 - u - 6 = 0$ $u = 3, u = -2$ $x = \ln 3 = 1.099,$ ~~$x = \ln -2$~~

$(u-3)(u-2) = 0$ $e^x = 3, e^x = -2$

6. $9^x - 3^{x+1} - 10 = 0$ $u^2 - 3u - 10 = 0$ $3^x = 5, 3^x = -2$

$3^{2x} - 3^{x+1} - 10 = 0$ $(u-5)(u+2) = 0$ $x = \log_3 5$; ~~$x = \log_{-3} -2$~~

$3^{2x} - 3^x \cdot 3^1 - 10 = 0$ $u = 5, u = -2$

7. The function $f(x) = A \cdot B^{3x}$ crosses through the points $(0.5,1.5)$ and $(1,13.5)$. Find the values of A and B.

(1) $1.5 = A \cdot 3^{B(0.5)}$ $1.5 = A \cdot 3^2$ $A = \frac{1}{6}$

(2) $13.5 = A \cdot 3^{B(1)}$ $13.5 = A \cdot 3^4$

(2÷1) $9 = 3^{B(1-0.5)}$

$3^2 = 3^{B(0.5)}$

$2 = \frac{1}{2}B$

$B = 4$