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Lesson 3.8 - Solving Logarithmic Equations

I. Warm-Up: Condense the logarithmic expressions:

1. $4\log_9(x) - \left(\log_{\sqrt{3}}(y) + \log_{\frac{1}{3}}(z)\right)$

$\log_9 x^4 - \left(\log_{\sqrt{3}}(y) + \log_{\frac{1}{3}}(z)\right)$

$\frac{\log_3 x^4}{\log_3 9} - \frac{\log_3 y}{\log_3 \sqrt{3}} - \frac{\log_3 z}{\log_3 \frac{1}{3}}$

$\frac{\log_3 x^4}{2} - \frac{\log_3 y}{\frac{1}{2}} - \frac{\log_3 z}{-1}$

$\frac{4\log_3 x}{2} - \frac{\log_3 y}{\frac{1}{2}} + \frac{\log_3 z}{-1}$

$2\log_3 x - 2\log_3 y - \log_3 z$

$\log_3 x^2 - \log_3 y^2 - \log_3 z$

$\log_3 \frac{x^2}{y^2 z}$

II. Solving Log Equations

2. $\log_2(x) + \log_2(x+2) = \log_2(x+6)$

$\log_2(x(x+2)) = \log_2(x+6)$

$x^2 + 2x = x + 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

~~$x = -3$~~, $x = 2$

3. $\ln(x+1) - \ln(x-2) = \ln(x)$

$\ln(\frac{x+1}{x-2}) = \ln x$

$x^2 - 2x = x + 1$

$x^2 - 3x - 1 = 0$

$x = \frac{3 \pm \sqrt{13}}{2}$

4. $\log_3(x) - \log_{\frac{1}{3}}(x-9) = 2(\log_3(2) + 1)$

$\log_3 x - \frac{\log_3 x - 9}{\log_3 \frac{1}{3}} = 2(\log_3 2 + \log_3 3)$

$\log_3 x - \frac{\log_3 x - 9}{-1} = 2(\log_3 6)$

$\log_3(x\,(x-9)) = \log_3 6^2$

$\log_3(x^2 - 9x) = \log_3 36$

$x^2 - 9x = 36$

$x^2 - 9x - 36 = 0$

$(x-12)(x+3) = 0$

$x = 12, x \neq -3$

III. Harder Log Problems!

5. $\ln(x) + \ln(x+3) = 1$

$\ln(x) + \ln(x+3) = \ln e$ $\quad x^2 + 3x + e = 0$ $\quad x = \frac{-3 \pm \sqrt{9-4e}}{2}$

6. $\ln\left(\sqrt{x-8}\right) = 5$

$\sqrt{x-8} = e^5$ $\quad x - 8 = e^{10}$ $\quad x = e^{10} + 8$

7. $\log_4(x) + \log_4(x-3) = 1$

$\log_4\left(x(x-3)\right) = 1$ $\quad x^2 - 3x - 4 = 0$ $\quad x = 4, \sout{x = -1}$

$\log_4(x^2 - 3x) = 1$ $\quad (x-4)(x+1) = 0$

8. $\log\left[\log\left(\frac{x}{8}\right)\right] = 0$

$10^0 = 1 = \log\left(\frac{x}{8}\right)$ $\quad 10^1 = \frac{x}{8}$ $\quad x = 80$

9. $2\log_3(x) = 2\log_3(2) + \log_3(3-x)$

$\log_3 x^2 = \log_3 2^2 + \log_3(3-x)$ $\quad \log_3 \frac{x^2}{12-4x} = 0$ $\quad x^2 + 4x - 12 = 0$

$\log_3 x^2 = \log_3(4(3-x))$ $\quad 3^0 = 1 = \frac{x^2}{12-4x}$ $\quad (x+6)(x-2) = 0$

$\log_3 x^2 - \log_3(12-4x) = 0$ $\quad x^2 = 12 - 4x$ $\quad \sout{x = -6}, x = 2$

10. $\log_{36}(x) + \frac{1}{4}\log_{\sqrt{6}}(x-5) = 1$

$\frac{\log_6 x}{\log_6 36} + \frac{\log_6 (x-5)^{1/4}}{\log_6 \sqrt{6}} = 1$

$\frac{\log_6 x}{2} + \frac{\log_6 (x-5)^{1/4}}{1/2} = 1$

$\frac{1}{2}\log_6 x + 2\log_6 (x-5)^{1/4} = 1$

$\log_6 x^{1/2} + \log_6 (x-5)^{1/2} = 1$

$\log_6 x^{1/2} + \log_6 (x-5)^{1/2} = 1$

$\log_6 \sqrt{x^2 - 5x} = 1$

$6 = \sqrt{x^2 - 5x}$

$x^2 - 5x - 36 = 0$

$(x-9)(x+4) = 0$

$x = 9, \sout{x = -4}$