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Lesson 3.9 - Logarithmic & Exponential Models I

1. Francis Finklestein II invests $2000 into an account at an interest rate of 10%. Find how long it will take her to have $3000 if his investment is compounded

a. annually

$A = P(1 + \frac{r}{n})^{nt}$

$3000 = 2000(1 + \frac{0.1}{1})^{1t}$

$\frac{3}{2} = (1.1)^t$

$\log_{1.1} \frac{3}{2} = t$

$t = 4.254$ years

b. monthly

$3000 = 2000(1 + \frac{0.1}{12})^{12t}$

$\frac{3}{2} = (1 + \frac{0.1}{12})^{12t}$

$\log_{1+\frac{0.1}{12}} \frac{3}{2} = 12t$

$\frac{1}{12} \log_{1+\frac{0.1}{12}} \frac{3}{2} = t$

$t = 4.073$ years

c. continuously

$3000 = 2000e^{0.1t}$

$\frac{3}{2} = e^{0.1t}$

$\ln\left(\frac{3}{2}\right) = 0.1t$

$\frac{\ln\left(\frac{3}{2}\right)}{0.1} = t$

$t = 4.05$ years

2. Duplica invests $2000 at an annual rate of 5% and it is compounded daily.

a. Find the time it takes for her money to double.

$4000 = 2000(1 + \frac{0.05}{365})^{365t}$

$2 = (1 + \frac{0.05}{365})^{365t}$

$\log_{1+\frac{0.05}{365}} 2 = 365t$

$\frac{\log_{1+\frac{0.05}{365}} 2}{365} = t$

$t = 13.57$ years

b. Find the total amount of money she has after 10 years.

$A = 2000(1 + \frac{0.05}{365})^{3650} = \$3297.33$

3. Mr. Braza invests $1500 and it takes 12 years for his money to double if his money is compounded continuously.

a. Find the annual interest rate.

$3000 = 1500e^{r(12\ yr)}$ $\quad 2 = e^{r(12\ yr)}$ $\quad \ln 2 = 12r$ $\quad r = \frac{\ln 2}{12} = 0.0578 = 5.78\%$

b. Find the total amount of money he has after 15 years.

$A = \$1500e^{0.0578*15yr} = \$3567.61$

Radioactive Decay & Carbon Dating: What is half-life? Half life is the amount of time it takes for a substance to decay to half its value.

4. Carbon-14 has a half-life of 5715 years. If I begin with an initial quantity of 5g, how much will I have after 1000 years?

$$A = P(1 + \frac{r}{n})^{nt} = 5(\frac{1}{2})^{\frac{1000}{5715}} = 4.4289g$$

5. Carbon-14 has a half-life of 5715 years. Researchers detected 3.0 grams of Carbon-14 in a fossil. How much Carbon-14 was present in the fossil 1000 years ago?

$$A = P(1 + \frac{r}{n})^{nt} = P(\frac{1}{2})^{\frac{1000}{5715}} = 3 \qquad P = 3.3868g$$

6. Plutonium-239 has a half-life of 24,100 years. I store 2,500 grams in a container.

a. How much will I have after 10,000 years?

$$A = P(1 + \frac{r}{n})^{nt} = 2500(\frac{1}{2})^{\frac{10000}{24100}} = 1875.1$$

b. How long will it take to decay to 200 grams?

$$A = P(1 + \frac{r}{n})^{nt} = 200 = 2500(\frac{1}{2})^{\frac{t}{24100}}$$

$$0.08 = (\frac{1}{2})^{\frac{t}{24100}}$$

$$\log_{0.5}(0.08) * 24100 = t = 87816.93$$