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Lesson 5.3 – Double Angle Formula – Trig. Equations

I. Deriving the Double Angle Formulas

1. Find $\sin(2\theta)$ using $\sin(\theta+\theta)$

$\sin\theta\cos\theta + \cos\theta\sin\theta$

$2\sin\theta\cos\theta$

2. Find $\cos(2\theta)$ using $\cos(\theta+\theta)$

$\cos\theta\cos\theta - \sin\theta\sin\theta$

$\cos^2\theta - \sin^2\theta$

$(1-\sin^2\theta) - \sin^2\theta$ $\quad$ $\cos^2\theta - (1-\cos^2\theta)$

$1 - 2\sin^2\theta$ $\quad$ $2\cos^2\theta - 1$

II. Using Double Angle Formulas to Solve Trig. Equations

Solve for θ in the interval $[0, 2\pi)$

Double Angle Formulas

$$\sin 2u = 2\sin u\cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 1 - 2\sin^2 u$$

$$\cos 2u = 2\cos^2 u - 1$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

3. $2\sin^2\theta + \sin(2\theta) = 0$

$2\sin^2\theta + 2\sin\theta\cos\theta = 0$

$2\sin\theta(\sin\theta + \cos\theta) = 0$

$2\sin\theta = 0$ and $\sin\theta + \cos\theta = 0$

$\theta = 0, \pi$ and $\sin\theta = -\cos\theta$

$\tan\theta = -1$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

Which formula of cosine should you use and why?

4. $\cos(2\theta) + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$2\cos\theta(\cos\theta + 1) - (\cos\theta + 1) = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ $\quad$ $\cos\theta = -1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ $\theta = \pi$

5. $4\sin^2\theta - \sin\theta = \cos(2\theta)$ [*Use Calculator*]

$4\sin^2\theta - \sin\theta = 1 - 2\sin^2\theta$

$6\sin^2\theta - \sin\theta - 1 = 0$

$6\sin^2\theta - 3\sin\theta + 2\sin\theta - 1 = 0$

$3\sin\theta(2\sin\theta - 1) + 1(2\sin\theta - 1) = 0$

$(3\sin\theta + 1)(2\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{3}$ $\quad$ $\sin\theta = \frac{1}{2}$

$\arcsin(-\frac{1}{3}) = \theta$ for $\quad$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(Quadrant III and IV)

$199.47°, 340.53°$

III. **Practice** – Solve for x or θ in the interval $[0,2\pi)$.

6. $\cos^2\theta = \frac{1}{2}\sin(2\theta)$

$\cos^2\theta = \frac{1}{2}(2\sin\theta\cos\theta)$

$\cos^2\theta = \sin\theta\cos\theta$

$\cos^2\theta - \sin\theta\cos\theta = 0$

$\cos\theta\,(\cos\theta - \sin\theta) = 0$

$\cos\theta = 0;$ $\quad\quad$ $\cos\theta - \sin\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad\quad$ $\cos\theta = \sin\theta$

$1 = \tan\theta$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

7. $\cos(2\theta) + \sin^2\theta = 0$

$\cos^2\theta - \sin^2\theta + \sin^2\theta = 0$

$\cos^2\theta = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

8. $10\cos^2 x - 5\cos x - 1 = -\cos(2x)$ $\quad\quad$ [*Use Calculator*]

$10\cos^2 x - 5\cos x - 1 = -(2\cos^2 x - 1)$

$10\cos^2 x - 5\cos x - 1 = 1 - 2\cos^2 x$

$12\cos^2 x - 5\cos x - 2 = 0$

$12\cos^2 x - 8\cos x + 3\cos x - 2 = 0$

$4\cos x\,(3\cos x - 2) + 1(3\cos x - 2) = 0$

$(4\cos x + 1)(3\cos x - 2) = 0$

$\cos x = -\frac{1}{4}$ $\quad\quad$ $\cos x = \frac{2}{3}$

$x = 104.5°; 255.5°$ $\quad\quad$ $x = 48.2°, 311.8°$

9. $4\sin x\cos x - \sqrt{3} = 0$

$2(2\sin x\cos x) = \sqrt{3}$

$2\sin 2x = \sqrt{3}$

$\sin 2x = \frac{\sqrt{3}}{2}$

$2x = \sin^{-1}\frac{\sqrt{3}}{2}$

$2x = \frac{\pi}{3}; \frac{2\pi}{3}$

$x = \frac{\pi}{6}; \frac{\pi}{3}$