Name: _____Kevin Braza – KEY ___________________________ Date: _________________ IB Math A&A SL

Lesson 7.4 – Double Angle Formula – Other Applications and Proofs

I. Warm-Up

1. Verify the following identity: $\cot x - \tan y = \frac{\cos(x+y)}{\sin x \cos y}$

$$\frac{\cos x}{\sin x} - \frac{\sin y}{\cos y} = \frac{\cos(x+y)}{\sin x \cos y}$$

$$\frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y} = \frac{\cos(x+y)}{\sin x \cos y}$$

$$\frac{\cos(x+y)}{\sin x \cos y} = \frac{\cos(x+y)}{\sin x \cos y}$$

2. Let $\tan x = -\frac{3}{4}$ and x exists in Quadrant IV. Find $\sin x$ and $\cos x$.

$\sin x = -\frac{3}{5}$; $\cos x = \frac{4}{5}$

II. Using Triangles in Double Angle Formula

3. Use the results from #2. Use the double angle formulas to find:

a. $\sin(2x)$

$2 \sin x \cos x$

$2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$-\frac{24}{25}$

b. $\cos(2x)$

$\cos^2 x - \sin^2 x$

$\left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$

$\frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

c. $\tan^2(2x)$

$\frac{\frac{-24}{25}}{\frac{7}{25}} = -\frac{24}{7}$

4. Let $\sin\theta = \frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$

a. Find $\sin(2\theta)$

$2 \sin\theta \cos\theta$

$2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$

$\frac{-120}{169}$

b. Find $\cos(2\theta)$

$2\cos^2\theta - 1$

$2\left(-\frac{12}{13}\right)^2 - 1$

$2\left(\frac{144}{169}\right) - \frac{169}{169}$

$\frac{288}{169} - \frac{169}{169}$

$\frac{119}{169}$

c. Find $\tan(2\theta)$

$\frac{\frac{-120}{169}}{\frac{119}{169}} = -\frac{120}{119}$

III. Trig Proofs with Double Angle Formulas

Verify the following identities.

5. $\sin(2x) = \tan(x)(1 + \cos(2x))$

$= \frac{\sin x}{\cos x}(1 + 2\cos x - 1)$

$\frac{= \sin x (2\cos x)}{\cos x}$

$= 2\sin x \cos x$

$= \sin(2x)$

6. $\sin(3x) = 3\sin x - 4\sin^3 x$

$\sin(2x)\cos x + \cos(2x)\sin x$

$2\sin x\cos^2 x + (1 - 2\sin^2 x)\sin x$

$2\sin x\cos^2 x + \sin x - 2\sin^3 x$

$2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$3\sin x - 4\sin^3 x$

7. $\cos^2 x = \frac{1}{2}(\cos(2x) + 1)$

$\frac{1}{2}((2\cos^2 x - 1) + 1)$

$\frac{1}{2}((2\cos^2 x))$

$\cos^2 x$

8. $\sec(2\theta) = \frac{\sec^2(\theta)}{2 - \sec^2(\theta)}$

$\frac{1}{\cos(2\theta)}$

$\frac{1}{2\cos^2\theta - 1}$

$\frac{1}{\frac{2}{\sec^2\theta} - 1}$

$\frac{1}{\frac{2 - \sec^2\theta}{\sec^2\theta}}$

$\frac{\sec^2\theta}{2 - \sec^2\theta}$

9. $\cos(4x) = 8\cos^4 x - 8\cos^2 x + 1$

$2\cos^2(2x) - 1$

$2(\cos 2x)(\cos 2x) - 1$

$2(2\cos^2 x - 1)(2\cos^2 x - 1) - 1$

$2(4\cos^4 x - 4\cos^2 x + 1) - 1$

$8\cos^4 x - 8\cos^2 x + 2 - 1$

$8\cos^4 x - 8\cos^2 x + 1$