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Lesson 8.1 – A More Formal Intro to Vectors

I. Using your Trigonometry: An airplane in calm conditions is flying at 800 km/hr due east. A cold wind suddenly blows from the south at 35 km/hr pushing the airplane slightly off course. Draw a picture of the scenario, and using trigonometry, find the resulting speed and direction on the plane

Angle = 0.044 rad or 2.5 degrees

What are vectors? **A vector is a quantity that has both a magnitude and a direction. Quantities that only have a magnitude are called scalars.**

II. Representing Vectors in the Coordinate Plane

1. Write each of the following vectors in unit vector form.

   a. $\begin{pmatrix}3\\4\end{pmatrix} = 3\boldsymbol{i} + 4\boldsymbol{j}$

   b. $\begin{pmatrix}2\\0\end{pmatrix} = 2\boldsymbol{i} + 0\boldsymbol{j}$

   c. $\begin{pmatrix}2\\-5\end{pmatrix} = 2\boldsymbol{i} - 5\boldsymbol{j}$

   d. $\begin{pmatrix}-1\\-3\end{pmatrix} = -\boldsymbol{i} - 3\boldsymbol{j}$

2. Find the unknowns if

   a. $\begin{pmatrix}a+1\\2b-8\end{pmatrix} = \begin{pmatrix}9-a\\a\end{pmatrix}$

$a + 1 = 9 - a$ $\quad 2a = 8$ $\quad a = 4$

$2b - 8 = 4$ $\quad 2b = 12$ $\quad b = 6$

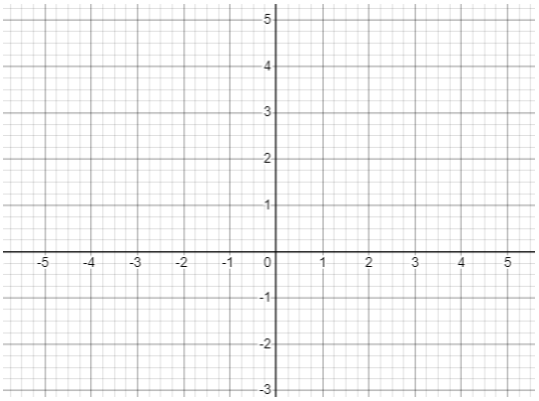
   b. $\begin{pmatrix}2x+3y\\x-2\end{pmatrix} = \begin{pmatrix}11\\2y\end{pmatrix}$

$2x + 3y = 11$ $\quad 2x + 3y = 11$ $\quad 2x + 3y = 11$

$x - 2 = 2y$ $\quad x - 2y = 2$ $\quad -2x + 4y = -4$

Add equations: $\quad 7y = 7$ $\quad y = 1$

$x - 2 = 2(1)$ $\quad x = 4$



- Plot the point (2,5).
- Plot the vector $< 2,5 >$ or $\begin{pmatrix}2\\5\end{pmatrix}$

Rewriting in terms of unit vectors:

Let $\boldsymbol{i} = \begin{pmatrix}\mathbf{1}\\\mathbf{0}\end{pmatrix}$ and $\boldsymbol{j} = \begin{pmatrix}\mathbf{0}\\\mathbf{1}\end{pmatrix}$

$$\begin{pmatrix}x\\y\end{pmatrix} = x\boldsymbol{i} + y\boldsymbol{j}$$

$$\begin{pmatrix}2\\5\end{pmatrix} = 2\boldsymbol{i} + 5\boldsymbol{j}$$

III. The Magnitude & Direction of a Vector

3. Given the following information, find the component form & unit vector form of each vector, as well as the magnitude and direction angle for each vector.

a. $\overrightarrow{RS}$ where $R = (7,2)\,; S = (-1,-10)$

Component Form: $\begin{pmatrix}-8\\-12\end{pmatrix}$ Unit Vector Form: $-8\boldsymbol{i} - 12\boldsymbol{j}$

Magnitude: $\left|\overrightarrow{RS}\right| = \sqrt{(-8)^2 + (-12)^2} = 4\sqrt{13} \approx 14.422$

$\arctan\left(\frac{-12}{-8}\right) = 56.31°$, Angle in 3rd Quadrant = $180 + 56.31°$

Direction: 236.31°

b. $\overrightarrow{PQ}$ where $P = (-4,-10)\,; Q = (-5,-2)$

Component Form: $\begin{pmatrix}-1\\12\end{pmatrix}$ Unit Vector Form: $\boldsymbol{i} + 12\boldsymbol{j}$

Magnitude: $\left|\overrightarrow{PQ}\right| = \sqrt{(-1)^2 + (12)^2} = \sqrt{145} \approx 12.042$

$\arctan\left(\frac{12}{1}\right) = 85.23°$ Angle in 2nd Quadrant = $180 - 85.24°$

Direction: 94.77°

c. $\overrightarrow{RS}$ where $R = (10,7)\,; S = (-5,-3)$

Component Form: $\begin{pmatrix}-15\\-10\end{pmatrix}$ Unit Vector Form: $-15\boldsymbol{i} - 10\boldsymbol{j}$

Magnitude: $\left|\overrightarrow{RS}\right| = \sqrt{(-15)^2 + (-10)^2} = 5\sqrt{13} \approx 18.028$

$\arctan\left(\frac{-10}{-15}\right) = 33.69°$, Angle in 3rd Quadrant = $180 + 33.69°$

Direction: 213.69°

d. $\overrightarrow{RS}$ where $R = (-6,-4)\,; S = (-8,-7)$

Component Form: $\begin{pmatrix}-2\\-3\end{pmatrix}$ Unit Vector Form: $-2\boldsymbol{i} - 3\boldsymbol{j}$

Magnitude: $\left|\overrightarrow{RS}\right| = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13} \approx 3.606$

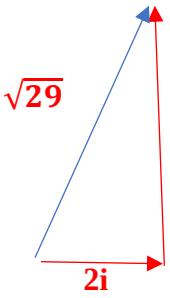
$\arctan\left(\frac{-3}{-2}\right) = 56.31°$, Angle in 3rd Quadrant = $180 + 56.31°$

Direction: 236.31°

Consider our previous vector: $\begin{pmatrix}2\\5\end{pmatrix}$

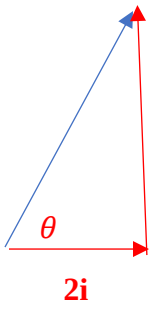
Which can be written in base unit vector form as $2\boldsymbol{i} + 5\boldsymbol{j}$.

The **length** or **magnitude** of the vector is the distance of the vector from the standard position in the origin (This is just the Pythagorean Theorem).



$|\boldsymbol{a}| = \sqrt{{a_1}^2 + {a_2}^2}$

The **direction** of a vector is the measure of the angle it makes with a horizontal line. Using trigonometry,



$\tan\theta = \frac{y}{x}$

4. Given the magnitude and direction, find the horizontal and vertical components for the following vectors.

   a. $|\boldsymbol{a}|, \boldsymbol{\theta} = 45, 298°$

$x = 45 \cos 298° = 21.13$

$y = 45 \sin 298° = -39.73$

   b. $|\boldsymbol{m}|, \boldsymbol{\theta} = 17, 41°$

$x = 17 \cos 41° = -15.17$

$y = 17 \sin 41° = -11.43$

   c. $|\boldsymbol{a}|, \boldsymbol{\theta} = 11, 99°$

$x = 11 \cos 99° = -1.72$

$y = 11 \sin 99° = 10.86$

   d. $|\boldsymbol{t}|, \boldsymbol{\theta} = 17, 41°$

$x = 17 \cos 41° = 12.83$

$y = 17 \sin 41° = 11.15$