Name: ____ Date: ______________ IB Math A&A SL

Unit 2 Test – *No Calculators Allowed (Practice Version)*

Show all your work. Indicate clearly the methods you use, because you will be graded on correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. Consider the function below:

$$f(x) = -\frac{x^2}{2} - 4x - 5$$

a. Complete the square for the function.

$f(x) = -\frac{1}{2}(x^2 + 8x) - 5$

$f(x) = -\frac{1}{2}(x^2 + 8x\ [+16 - 16]) - 5$

$f(x) = -\frac{1}{2}((x+4)^2 - 16) - 5$

$f(x) = -\frac{1}{2}(x+4)^2 + 8 - 5$

$f(x) = -\frac{1}{2}(x+4)^2 + 3$

b. Identify the vertex. State if the vertex is a maximum or a minimum. Vertex: $(-4,3)$ Maximum

c. Identify the axis of symmetry. $x = -4$

d. Find the discriminant and determine how many roots this function has.
$(-4)^2 - 4\left(-\frac{1}{2}\right)(-5) = 16 + 2(-5) = 6$ →positive, 2 real roots

e. Identify any x-intercepts.

$0 = -\frac{1}{2}(x+4)^2 + 3$

$-3 = -\frac{1}{2}(x+4)^2$

$6 = (x+4)^2$

$\pm\sqrt{6} = x + 4$

$x = -4 \pm \sqrt{6}$

x-intercepts: $\left(-4 \pm \sqrt{6}, 0\right)$

f. Identify any y-intercepts.

$y = -\frac{0^2}{2} - 4(0) - 5 = -5$

y-intercept: $(0, -5)$

2. Complete the following:

a. Find the equation of a parabola that has the vertex (2, 3) and passes through the point (0, 2).

$y = a(x-h)^2 + k$

$y = a(x-2)^2 + 3$

$2 = a(0-2)^2 + 3$

$-1 = a(4)$

$a = -\frac{1}{4}$

$y = -\frac{1}{4}(x-2)^2 + 3$

b. Rewrite the equation from part a in standard form.

$y = -\frac{1}{4}(x-2)^2 + 3$

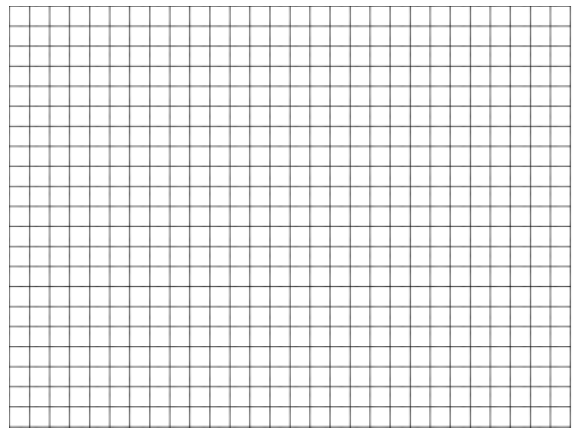
$y = -\frac{1}{4}(x^2 - 4x + 4) + 3$

$y = -\frac{1}{4}x^2 + x - 1 + 3$

$y = -\frac{1}{4}x^2 + x + 2$

c. Given a parent function $f(x) = x^2$, state the transformations from $f(x)$ to the function in part a.

- Reflection across the x-axis
- Vertical compression by a factor of 4
- Horizontal shift by 2 to the right
- Vertical shift by 3 up



d. Sketch a graph of part a.

Check on Desmos.

3. Mr. Braza has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fencing along the river. What are the dimensions of the field that has the largest area?

$P = l + 2w = 2400 \rightarrow l = 2400 - 2w$

$A = lw = (2400 - 2w)w = 2400w - 2w^2$

Axis of symmetry: $w = \frac{-2400}{2(-2)} = \frac{2400}{4} = 600$

$l = 2400 - 2(600) = 1200$

4. Derive the quadratic formula given a quadratic equation of the form $ax^2 + bx + c = 0$.

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x\,[+\left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2] + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2} = 0$

$(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a^2} = 0$

$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

5. **IB Math A&A SL Summer 21/Zone 2 - Paper 2, Question #5 [Maximum mark: 6]**

No calculator this time, but here's some hints.

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$ where $c \in \mathbb{R}$.

Hint #1: Rewrite $f(x)$ in vertex form. Determine whether the vertex is a maximum or minimum.

$f(x) = 6x^2 - 12x + 1$

$f(x) = 6(x^2 - 2x) + 1$

$f(x) = 6(x^2 - 2x\ [+1 - 1]) + 1$

$f(x) = 6((x-1)^2 - 1) + 1$

$f(x) = 6(x-1)^2 - 6 + 1$

$f(x) = 6(x-1)^2 - 5$

a. Find the range of f.

$\{y | y \geq -5\}$ The vertex is a minimum so the parabola includes all values of y above and equal to -5.

Hint #2: Evaluate $(g \circ f)(x)$. Do not simplify.

$(g \circ f)(x) = -(6x^2 - 12x + 1) + c$

Hint #3: Discuss the transformations from $f(x)$ to $(g \circ f)(x)$.

- Reflection across the x-axis
- Vertical shift up or down by c.

Hint #4: Discuss what happens to the vertex & range of the function under the above transformations.

- Vertex moves from $(1, -5)$ to $(1, 5 + c)$ and changes to a maximum.
- Range changes to $\{y | y \leq 5 + c\}$

b. Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c.

If the range has changed to $y \leq 5 + c$ and the function has a restriction $(g \circ f)(x) \leq 0$, then $c \leq -5$ for the maximum point of the function to be less than or equal to zero.