Name: ____Kevin Braza – KEY__________________________ Date: __________________ IB Math A&A SL

Unit 3 Test – *Calculators Allowed, <u>but ALL work must be shown</u>. (Practice Version 1)*

Show all your work. Indicate clearly the methods you use, because you will be graded on correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. Let $f(x) = k\log_2 x$. Given that $f^{-1}(1) = 8$, find the value of k. Then find $f^{-1}\left(\frac{2}{3}\right)$.

$1 = k\log_2 8$

$1 = k(3)$

$k = \frac{1}{3}$

$\frac{2}{3} = \frac{1}{3}\log_2 x$

$2 = \log_2 x$

$x = 4$

2. Find the product of the zeroes of $g(x) = \log_5 |2\log_3 x|$

$0 = \log_5 |2\log_3 x|$

$5^0 = |2\log_3 x|$

$1 = |\log_3 x^2|$

$\pm 1 = \log_3 x^2$

$\log_3 x^2 = 1$ and $\log_3 x^2 = -1$

$3^1 = x^2$ and $3^{-1} = x^2$

$x = \sqrt{3}$ and $x = \frac{1}{\sqrt{3}}$

Product = 1

Express as a single logarithm in its simplest form.

3. $\ln(x^2 - 1) - 2\ln(x+1) + \ln(x^2 + x)$

$\ln\left(\frac{(x^2-1)(x^2+x)}{(x+1)^2}\right) = \ln\left(\frac{\cancel{(x+1)}(x-1)(x)\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x+1)}}\right) = \ln(x^2 - x)$

4. $\log_4 8 - \log_4 4 + \log_4 32$

$\log_4\left(\frac{8 \cdot 32}{4}\right) = \log_4(2 \cdot 32) = \log_4(64) = 3$

5. Given the function: $f(x) = \log_3(x-2) + 1$

Identify the following properties:

a. Domain: $\{x | x > 2\}$
b. Range: $\{y | y \in \mathbb{R}\}$
c. Increasing/Decreasing: Check Desmos
d. Intercepts: x-int: $(\frac{7}{3}, 0)$; y-int: none
e. Asymptotes: VA: x=2
f. Find $f^{-1}(x)$ $y = \frac{1}{3}(3)^x + 2$
g. Graph both $f(x)$ and $f^{-1}(x)$
Check Desmos

Solve the following equations for all values of x.

6. $5^{x+3} = 19^x$

$\log 5^{x+3} = \log 19^x$

$(x+3)(\log 5) = x \log 19$

$x \log 5 + 3 \log 5 = x \log 19$

$x \log 5 - x \log 19 = -3 \log 5$

$x\,(\log 5 - \log 19) = -3 \log 5$

$x = \frac{=-3 \log 5}{\log 5 - \log 19} = \frac{3 \log 5}{\log 19/5}$

7. $\ln 2^{4x-1} = \ln 8^{x+5} + \log_2 16^{1-2x}$

$\ln 2^{4x-1} = \ln 2^{3(x+5)} + \log_2 2^{4(1-2x)}$

$\ln\left(\frac{2^{4x-1}}{2^{3x+15}}\right) = 4(1-2x)$

$\ln\left(\frac{2^{4x-1}}{2^{3x+15}}\right) = 4(1-2x)$

$\ln(2^{x-16}) = 4 - 8x$

$(x-16)\ln(2) = 4 - 8x$

$x \ln 2 - 16 \ln(2) = 4 - 8x$

$x \ln 2 + 8x = 16 \ln(2) + 4$

$x\,(\ln 2 + 8) = 16 \ln(2) + 4$

$x = \frac{16 \ln(2) + 4}{\ln 2 + 8}$

8. $\log_x 4 - \log_2 x = 1$

$\frac{\log_2 4}{\log_2 x} - \log_2 x = 1$

$2 - (\log_2 x)^2 = \log_2 x$

$(\log_2 x)^2 + \log_2 x - 2 = 0$

$(\log_2(x) + 2)(\log_2(x) - 1) = 0$

$\log_2(x) = -2$ and $\log_2(x) = 1$

$x = 2^{-2} = -\frac{1}{4}; x = 2^1 = 2$

9. $(1+\sqrt{2})^x + (1-\sqrt{2})^x = 6$

Guess and check, the answer is ± 2.

10. Solve the system of equations:

(1) $\log_{x+1} y = 2$ (2) $\log_{y+1} x = \frac{1}{4}$

$(x+1)^2 = y$

$(y+1)^{\frac{1}{4}} = x$

$y = x^4 - 1$

$(x+1)^2 = x^4 - 1$

~~$(x+1)$~~$(x+1) = (x-1)$~~$(x+1)$~~(x^2+1)

$x + 1 = (x-1)(x^2+1)$

$x + 1 = x^3 - x^2 + x - 1$

$0 = x^3 - x^2 - 2$

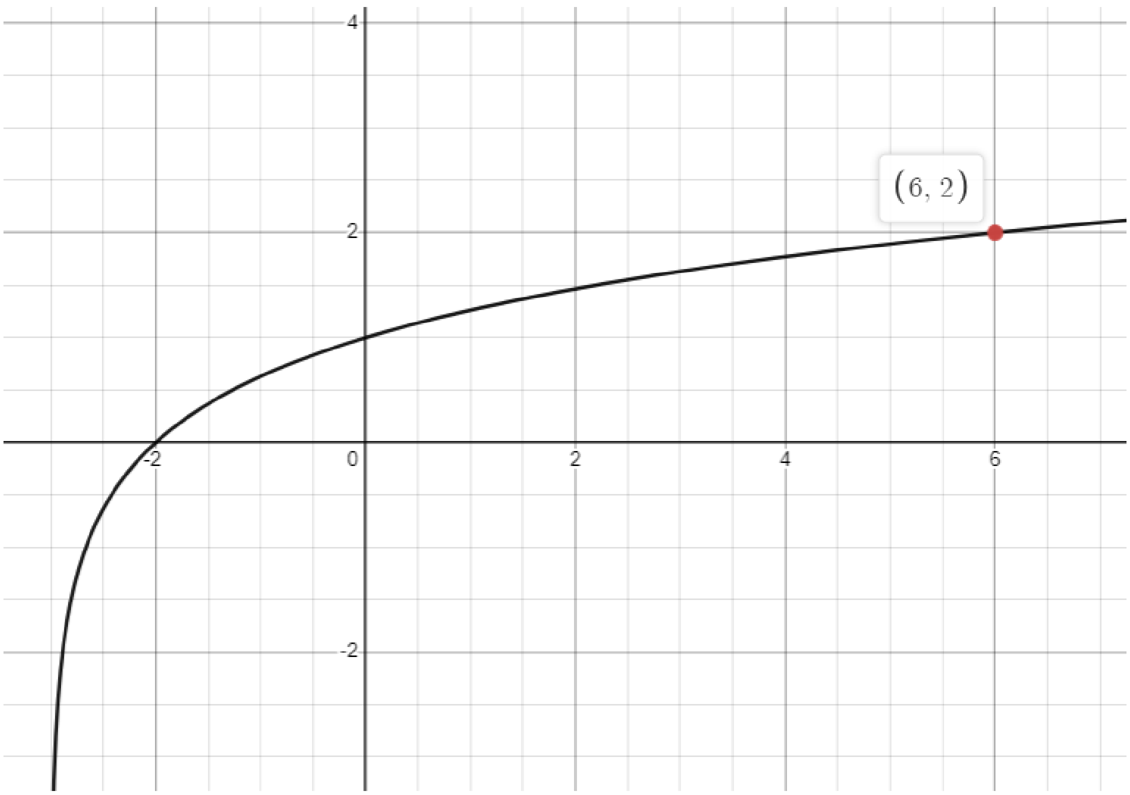
By calculator: $x = 1.69, y = 7.2$

I haven't shown you how to solve third degree polynomials (not in SL) so just practice with your calculator.

11. Radium-226 has a half-life of 1599 years. There is initially an amount of 7 grams present in a substance. How long will it take for it to decay to 2 grams?

$2 = 7(\frac{1}{2})^{t/1599}$ $t = 1599 \log_{1/2}(\frac{2}{7}) = 2889.96\ years$

12. Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below.



The graph passes through the point (6,2), has an x-intercept at (-2,0) and has an asymptote at $x = -3$. Find p.

$2 = \log_p 9$ $p^2 = 9$ $p = 3$

13. A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by $T = 280 \times 1.12^n$

At the end of 2000, there were 25600 people in the city who used taxis is given by:

$$P = \frac{2560000}{10 + 90e^{-0.1n}}$$

a. Find the value of P at the end of 2005, giving your answer to the nearest whole number.

$$\frac{2560000}{10 + 90e^{-0.1(5)}} = 39{,}636$$

b. After seven complete years, will the value of P double its value compared to the end of 2000? Justify your answer. No.

$$\frac{2560000}{10 + 90e^{-0.1(7)}} = 46{,}807$$

c. Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$. Find the value of R at the end of 2000.

$$R(0) = \frac{P(0)}{T(0)} = \frac{25600}{280} = 91.43$$

d. After how many complete years will the city first reduce the number of taxis?

Graph it and see after 10 complete years.