

Lesson 6.2B – Inverse Trigonometric Functions

I. Warm-Up

1. Find all values of θ in the interval $[0, 2\pi)$ such that:

a. $\sin \theta = \frac{1}{2}$

$\theta = 30^\circ \text{ or } 150^\circ$

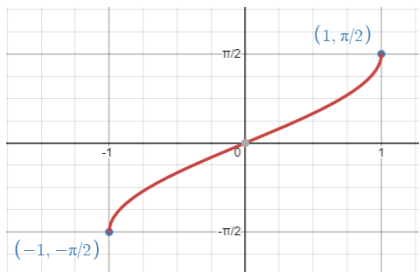
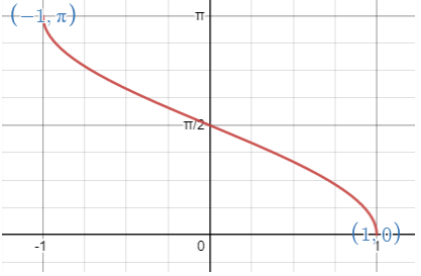
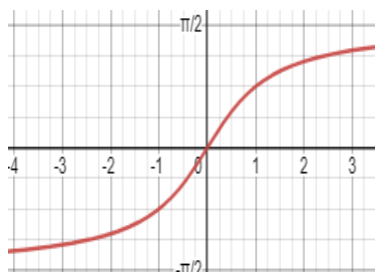
b. $\cos \theta = -\frac{\sqrt{2}}{2}$

$\theta = 135^\circ \text{ or } 225^\circ$

c. $\tan \theta = \frac{\sqrt{3}}{3}$

$\theta = 30^\circ \text{ or } 210^\circ$

II. Inverse Trigonometric Functions

<p>$y = \arcsin x$ or $y = \sin^{-1} x$</p>  <p>Domain: $[-1, 1]$ Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$</p>	<p>$y = \arccos x$ or $y = \cos^{-1} x$</p>  <p>Domain: $[-1, 1]$ Range: $[0, \pi]$</p>	<p>$y = \arctan x$ or $y = \tan^{-1} x$</p>  <p>Domain: $(-\infty, \infty) \mathbb{R}$ Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$</p>
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III. Working Backwards

Evaluate the following values without using a calculator.

2. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ \text{ or } \frac{\pi}{4}$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ \text{ or } \frac{\pi}{6}$

$\arctan(\sqrt{3}) = 30^\circ \text{ or } \frac{\pi}{6}$

3. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -45^\circ \text{ or } -\frac{\pi}{4}$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = -30^\circ \text{ or } -\frac{\pi}{6}$

$\arctan(\sqrt{3}) = -30^\circ \text{ or } -\frac{\pi}{6}$

IV. Practice

4. $\arcsin\left(\frac{\sqrt{3}}{2}\right) = -60^\circ \text{ or } -\frac{\pi}{3}$

$\sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ \text{ or } -\frac{\pi}{6}$

$\arctan(1) = 45^\circ \text{ or } \frac{\pi}{4}$

$$5. \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ \text{ or } \frac{\pi}{6}$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ \text{ or } -\frac{\pi}{6}$$

$$\arctan(\sqrt{3}) = 30^\circ \text{ or } \frac{\pi}{6}$$

$$6. \arctan\left(\frac{\sqrt{3}}{3}\right) = 30^\circ \text{ or } \frac{\pi}{6}$$

$$\arcsin(1) = 90^\circ \text{ or } \frac{\pi}{2}$$

$$\cos^{-1}(-1) = 180^\circ \text{ or } \pi$$

Draw a triangle and evaluate the following.

$$7. \sin(\cos^{-1}(\frac{3}{5})) = \frac{4}{5}$$

$$8. \sec(\arcsin(-\frac{5}{7})) = \frac{7}{2\sqrt{6}}$$

$$9. \tan\left(\arctan\left(\frac{\sqrt{3}}{3}\right)\right) = \frac{\sqrt{3}}{3}$$

10. If $f(x) = \cos x$ and $g(x) = \tan^{-1}(x)$. Find $f \circ g(-1)$

$$\cos(\tan^{-1}(-1)) = \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$