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Lesson 6.3 – Simplifying Trigonometric Identities

I. Warm-Up

1. Evaluate all of the trigonometric values. What do you notice?
   a. $\sin(30°)$ and $\cos(60°)$ $= 1/2$
   c. $\tan(60°)$ and $\cot(60°)$ $= \sqrt{3}$
   b. $\sec(45°)$ and $\csc(45°)$ $= \sqrt{2}$
   d. $\sin(15°)$ and $\cos(75°)$ (Use calculator) $= 0.258$

2. Which functions are even/odd?
   $f(x) = \sin x$ odd $f(x) = \cos x$ even $f(x) = \tan x$ odd

Cofunction Identities

$\sin(\frac{\pi}{2} - u) = \cos u$ $\cos(\frac{\pi}{2} - u) = \sin u$

$\tan(\frac{\pi}{2} - u) = \cot u$ $\cot(\frac{\pi}{2} - u) = \tan u$

$\sec(\frac{\pi}{2} - u) = \csc u$ $\csc(\frac{\pi}{2} - u) = \sec u$

Even/Odd Identities

$\sin(-u) = -\sin u$ $\csc(-u) = -\csc u$

$\cos(-u) = \cos u$ $\sec(-u) = \sec u$

$\tan(-u) = -\tan u$ $\cot(-u) = -\cot u$

II. Simplifying More Trig Identities

Prove the following identities are true. Only work on one side of the equation.

1. $\cot\left(\frac{\pi}{2} - x\right)\cos(x) = \sin x$

$\tan x \cos x = \sin x$

$\frac{\sin x}{\cos x}\cos x = \sin x$

$\sin x = \sin x$

2. $\sec\left(\frac{\pi}{2} - x\right)\tan(-x) = -\sec x$

$\sec\left(-(x - \frac{\pi}{2})\right) \cdot -\tan(x) = -\sec x$

$\sec\left(x - \frac{\pi}{2}\right) \cdot -\tan(x) = -\sec x$

$\csc(x) \cdot -\tan(x) = -\sec x$

$\frac{1}{\sin x} \cdot \frac{-\sin x}{\cos x} = -\sec x$

$\frac{-1}{\cos x} = -\sec x$

$-\sec x = -\sec x$

3. $\cos^4 x - 2\cos^2 x + 1 = \sin^4 x$

Let $u = \cos^2 x$

$u^2 - 2u + 1 = \sin^4 x$

$(u - 1)^2 = \sin^4 x$

$(\cos^2 x - 1)^2 = \sin^4 x$

$(-\sin^2 x)^2 = \sin^4 x$

$\sin^4 x = \sin^4 x$

4. $(2\csc x + 2)(2\csc x - 2) = 4\cot^2 x$

$4\csc^2 x - 4$

$4(\csc^2 x - 1)$

$4\cot^2 x$

III. Practice

Prove the following identities are true. Only work on one side of the equation.

5. $\tan x + \frac{\sec^2 x}{\tan(-x)} = \cot x$

$\frac{-\tan^2 x}{-\tan x} + \frac{\sec^2 x}{-\tan x} = -\cot x$

$\frac{\sec^2 x - \tan^2 x}{-\tan x} = -\cot x$

$\frac{1}{-\tan x} = -\cot x$

$-\cot x = -\cot x$

6. $\csc\left(\frac{\pi}{2} - x\right)\cot(-x) = -\csc x$

$-\sec x \cdot \cot x$

$-\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}$

$-\frac{1}{\sin x}$

$-\csc x$

7. $\frac{1}{\sec(x)+1} - \frac{1}{\sec(x)-1} = -2\cot^2 x$

$\frac{(\sec(x)-1)-(\sec(x)+1)}{(\sec(x)+1)(\sec(x)-1)} = -2\cot^2 x$

$\frac{-2}{\sec^2(x)-1} = -2\cot^2 x$

$\frac{-2}{\tan^2(x)} = -2\cot^2 x$

$-2\cot^2 x = -2\cot^2 x$

8. $\sec^3 x - \sec^2 x - \sec x + 1 = \tan^2 x(\sec x - 1)$

$\sec^2 x(\sec x - 1) - (\sec x - 1)$

$(\sec x - 1)(\sec^2 x - 1)$

$\tan^2 x(\sec x - 1)$

9. $\sec^4 x - \tan^4 x = 2\tan^2 x + 1$

$\frac{1}{\cos^4 x} - \frac{\sin^4 x}{\cos^4 x}$

$\frac{1-\sin^4 x}{\cos^4 x}$

$\frac{(1+\sin^2 x)(1-\sin^2 x)}{\cos^4 x}$

$\frac{(1+\sin^2 x)(\cos^2 x)}{\cos^4 x}$

$\frac{(1+\sin^2 x)}{\cos^2 x}$

$\frac{(\sin^2 x + \cos^2 x + \sin^2 x)}{\cos^2 x}$

$\frac{(2\sin^2 x + \cos^2 x)}{\cos^2 x}$

$2\tan^2 x + 1$

10. $\ln(\cos^2\theta) + \ln(1 + \tan^2\theta) = 0$

$\ln(\cos^2\theta(1 + \tan^2\theta)) = 0$

$\ln(\cos^2\theta(\sec^2\theta)) = 0$

$\ln(\cos^2\theta(\frac{1}{\cos^2\theta})) = 0$

$\ln 1 = 0$

$0 = 0$