

Lesson 6.4 – Proving Trigonometric Identities

I. Warm-Up

1. Show that $3^{x-2} = \frac{1}{9}(3^x)$

$$3^{x-2} = 3^{-2} \cdot 3^x$$

$$3^{x-2} = 3^{x-2}$$

1. Look at both sides & similarities

(Don't change similarities)

2. Simplify harder-looking side (one side only)

3. Sine/Cosine are most comfortable

II. Proving Trigonometric Identities: Show that the two trigonometric identities are equivalent.

2. Show that

$$\csc^2 \theta + \sec^2 \theta - \tan^2 \theta - \cot \theta = 2$$

$$\sec^2 \theta - \tan^2 \theta + \csc^2 \theta - \cot \theta = 2$$

$$1 + 1 = 2$$

$$2 = 2$$

3. Show that

$$\tan \theta + \cot \theta = \sec \theta \csc \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$\frac{1}{\cos \theta \sin \theta}$$

$$\frac{1}{\cos \theta \sin \theta}$$

$$\sec \theta \csc \theta$$

4. Show that $\frac{\cot^2 \theta}{1 + \csc \theta} = \frac{1 - \sin \theta}{\sin \theta}$

$$\frac{\csc^2 \theta - 1}{1 + \csc \theta}$$

$$\frac{1 + \csc \theta}{(1 + \csc \theta)(1 - \csc \theta)}$$

$$\frac{1 + \csc \theta}{1 + \csc \theta}$$

$$\csc \theta - 1$$

$$\frac{1}{\sin \theta} - \frac{\sin \theta}{\sin \theta}$$

$$\frac{1 - \sin \theta}{\sin \theta}$$

$$\frac{1 - \sin \theta}{\sin \theta}$$

$$\frac{1 - \sin \theta}{\sin \theta}$$

$$\frac{1 - \sin \theta}{\sin \theta}$$

$$\frac{1 - \sin \theta}{\sin \theta}$$

5. Show that $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 2 \sec^2 \theta$

$$\frac{1 + \sin \theta + 1 - \sin \theta}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$\frac{2}{1 - \sin^2 \theta}$$

$$\frac{2}{\cos^2 \theta}$$

$$2 \sec^2 \theta$$

$$2 \sec^2 \theta$$

III. Practice: Show that the two trigonometric identities are equivalent.

6. $\sec x - \cos x = \sin x \tan x$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x}$$

$$\sin x \tan x$$

7. $\frac{\sin \theta}{1+\cos \theta} + \frac{\cos \theta}{\sin \theta} = \csc \theta$

$$\frac{\sin^2 \theta + \cos \theta + \cos^2 \theta}{(1+\cos \theta) \sin \theta}$$

$$\frac{1+\cos \theta}{(1+\cos \theta) \sin \theta}$$

$$\frac{1+\cos \theta}{(1+\cos \theta) \sin \theta} = \csc \theta$$

8. $\frac{1}{\tan \beta} + \tan \beta = \frac{\sec^2 \beta}{\tan \beta}$

$$\frac{1}{\tan \beta} + \frac{\tan^2 \beta}{\tan \beta}$$

$$\frac{1+\tan^2 \beta}{\tan \beta}$$

$$\frac{\sec^2 \beta}{\tan \beta}$$

9. $\sec^2 x + \csc^2 x = (\sec^2 x)(\csc^2 x)$

$$\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right)$$

$$\left(\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}\right)$$

$$\left(\frac{1}{\cos^2 x \sin^2 x}\right)$$

$$\left(\frac{1}{\cos^2 x}\right) \left(\frac{1}{\sin^2 x}\right)$$

$$(\sec^2 x)(\csc^2 x)$$

10. $\cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$

$$\cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos^2 \theta - 1 + \cos^2 \theta$$

$$2 \cos^2 \theta - 1$$

11. $\frac{\sec x - 1}{1 - \cos x} = \sec x$

$$\frac{\frac{1}{\cos x} - 1}{1 - \cos x}$$

$$\frac{\frac{1 - \cos x}{\cos x}}{1 - \cos x}$$

$$\frac{1 - \cos x}{(1 - \cos x) \cos x}$$

$$\frac{1}{\cos x}$$

$$\sec x$$