

Lesson 6.5 – Practice with Proving Trigonometric Identities

I. Verify Each Identity (Work with each other)

$$\begin{aligned}
 1. \quad \sin^3 x \cos^4 x &= (\cos^4 x - \cos^6 x) \sin x \\
 &= \cos^4 x (1 - \cos^2 x) \sin x \\
 &= \cos^4 x (\sin^2 x) \sin x \\
 &= \sin^3 x \cos^4 x
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \sec^2 y - \cot^2\left(\frac{\pi}{2} - y\right) &= 1 \\
 \sec^2 y - \tan^2 y &= 1 \\
 1 &= 1
 \end{aligned}$$

$$3. (\sin^4 \beta - 2 \sin^2 \beta + 1) \cos \beta = \cos^5 \beta$$

$$4. \frac{1+\sin \theta}{\cos \theta} + \frac{\cos \theta}{1+\sin \theta} = 2 \sec \theta$$

$$(2 \sin^2 \beta - 1)^2 \cos \beta =$$

$$\frac{(1+\sin \theta)^2 + \cos^2 \theta}{\cos \theta(1+\sin \theta)}$$

$$(-(1 - 2 \sin^2 \beta))^2 \cos \beta =$$

$$\frac{1+2 \sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta(1+\sin \theta)}$$

$$-(\cos^2 \beta))^2 \cos \beta =$$

$$\frac{2+2 \sin \theta}{\cos \theta(1+\sin \theta)}$$

$$\cos^4 \beta \cos \beta =$$

$$\frac{2(1+\sin \theta)}{\cos \theta(1+\sin \theta)}$$

$$\cos^5 \beta =$$

$$\frac{2}{\cos \theta} = \sec \theta$$

$$\begin{aligned}
 5. \quad \frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} &= 0 \\
 \frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x - \sin y)(\cos x + \cos y)} &= \\
 \frac{\cos^2 x + \sin^2 x - \sin^2 y - \cos^2 y}{(\sin x - \sin y)(\cos x + \cos y)} &= \\
 \frac{1 - 1}{(\sin x - \sin y)(\cos x + \cos y)} &= \\
 0 &
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \cos x - \frac{\cos x}{1 - \tan x} &= \frac{\sin x \cos x}{\sin x - \cos x} \\
 \frac{\cos x(1 - \tan x) - \cos x}{1 - \tan x} &= \\
 \frac{\cos x - \sin x - \cos x}{1 - \frac{\sin x}{\cos x}} &= \\
 \frac{-\sin x}{\frac{\cos x - \sin x}{\cos x}} &= \\
 \frac{-\sin x \cos x}{\cos x - \sin x} &= \\
 \frac{\sin x \cos x}{\sin x - \cos x} &
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} &= \frac{1 - \cos \theta}{|\sin \theta|} \\
 \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \cdot \sqrt{\frac{1 - \cos \theta}{1 - \cos \theta}} &= \\
 \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} &= \\
 \frac{1 - \cos \theta}{\sqrt{\sin^2 \theta}} &= \\
 \frac{1 - \cos \theta}{\sin \theta} &
 \end{aligned}$$

$$8. \sin^4 x + \cos^4 x = 1 - 2 \cos^2 x + 2 \cos^4 x$$

$$(1 - \cos^2 x)^2 + \cos^4 x =$$

$$1 - 2 \cos^2 x + \cos^4 x + \cos^4 x$$

$$1 - 2 \cos^2 x + 2 \cos^4 x$$