Name: _____Kevin Braza – KEY _____ Date: _____ IB Math A&A SL

Lesson 6.6 – Solving Trigonometric Equations I

I. Warm-Up:

1. Find all value of θ that satisfies the equation in the interval: $0 \le \theta \le 2\pi$.

a. $\cos\theta = -\frac{1}{2}$

Cos is negative in

Q2 and Q3

$Q2: \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$Q3: \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

b. $\tan\theta = \sqrt{3}$

Tan is positive in

Q1 and Q3

$Q1: \frac{\pi}{3}$

$Q3: \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

c. $\sin\theta = 0$

$0, \pi$

**How does your solution differ if I didn't give you a range restriction? **

II. Solve for all values of θ.

2. $5\sin\theta - 3\sin\theta - 1$

$2\sin\theta = -1$

Sin is negative in

Q3 and Q4

$\sin\theta = \pi + \frac{\pi}{6}$

$\sin\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{7\pi}{6} + 2\pi n$

$\theta = \frac{11\pi}{6} + 2\pi n$

3. $3\tan^2\theta - 1 = 0$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

All Quadrants

$\theta = \frac{\pi}{6} + 2\pi n$

$\theta = \frac{5\pi}{6} + 2\pi n$

$\theta = \frac{7\pi}{6} + 2\pi n$

$\theta = \frac{11\pi}{6} + 2\pi n$

4. $\cot\theta\cos^2\theta = 2\cot\theta$

$\cot\theta(\cos^2\theta - 2) = 0$

$\cot\theta = 0$

$\theta = \frac{\pi}{2}; \frac{3\pi}{2}$

5. $2\sin^2\theta + 3\sin\theta + 1 = 0$

$2\sin^2\theta + 2\sin\theta + \sin\theta + 1 = 0$

$2\sin\theta(\sin\theta + 1) + 1(\sin\theta + 1) = 0$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$

$2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6} + 2\pi n$

$\theta = \frac{11\pi}{6} + 2\pi n$

$\sin\theta + 1 = 0$

$\sin\theta = -1$

$\theta = \frac{3\pi}{2} + 2\pi n$

III. Practice

5. $3\sec^2\theta - 2\tan^2\theta = 4$

$3(\tan^2\theta + 1) - 2\tan^2\theta = 4$

$3\tan^2\theta + 3 - 2\tan^2\theta = 4$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\theta = \frac{\pi}{4} + \pi n$

$\theta = \frac{3\pi}{4} + \pi n$

6. $\sin\theta + \sqrt{2} = -\sin\theta$

$2\sin\theta = -\sqrt{2}$

$\sin\theta = \frac{-\sqrt{2}}{2}$

$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4} + 2\pi n$

$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} + 2\pi n$

7. $4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \frac{\pm\sqrt{3}}{2}$

$\theta = \frac{\pi}{3} + \pi n$

$\theta = \frac{2\pi}{3} + \pi n$

8. $\sin^2\theta = 2\sin\theta$

$\sin\theta\,(\sin\theta - 2) = 0$

$\sin\theta = 0$

$\theta = 0 + \pi n$

9. $2\cos^2\theta - 1 = \sin\theta$

$2(1 - \sin^2\theta) - 1 = \sin\theta$

$2 - 2\sin^2\theta - 1 = \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

$2\sin\theta\,(\sin\theta + 1) - 1(\sin\theta + 1) = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$2\sin\theta - 1 = 0$ $\qquad$ $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\sin\theta = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6} + 2\pi n$ $\qquad$ $\theta = \frac{3\pi}{2}$

10. $4\sin^3\theta + 2\sin^2\theta - 2\sin\theta - 1 = 0$

$2\sin^2\theta\,(2\sin\theta + 1) - (2\sin\theta + 1) = 0$

$(2\sin^2\theta - 1)(2\sin\theta + 1) = 0$

$2\sin^2\theta - 1 = 0$ $\qquad$ $2\sin\theta + 1 = 0$

$\sin^2\theta = \frac{1}{2}$ $\qquad$ $\sin\theta = -\frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$ $\qquad$ $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6} + 2\pi n$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$ $\qquad$ $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} + 2\pi n$

$\theta = \frac{\pi}{4} + \pi n$

$\theta = \frac{3\pi}{4} + \pi n$