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Lesson 6.7 Solving Trigonometric Equations II

I. Multiple Angle Trigonometric Equations

Find all solutions of x in $[0,2\pi)$. You may express your answers in degrees or radians.

1. $\cos(2x) = -\frac{1}{2}$

$2x = 120° \text{ and } 240°$

$\text{or } 2x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = 60° \text{ and } 120°$

$\text{or } x = \frac{\pi}{3}, \frac{2\pi}{3}$

2. $2\sin(2x) = -\sqrt{2}$

$\sin(2x) = -\frac{\sqrt{2}}{2}$

$2x = 225° \text{ and } 325°$

$\text{or } 2x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = 112.5° \text{ and } 162.5°$

$\text{or } x = \frac{5\pi}{8}, \frac{7\pi}{8}$

3. $2\cos(3x) - 1 = 0$

$\cos(3x) = \frac{1}{2}$

$3x = 60° \text{ and } 300°$

$\text{or } 3x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = 20° \text{ and } 100°$

$\text{or } x = \frac{\pi}{9}, \frac{5\pi}{9}$

II. Approximate Solutions for Trigonometric Equations

Find all solutions of x in $[0,2\pi)$. You may express your answers in degrees or radians.

4. $3\sin x - 2 = 0$

$\sin x = \frac{2}{3}$

$x = 41.8° \text{ and } 138.2°$

$\text{or } x = 0.729 \text{ rad and } 2.41 \text{ rad}$

5. $4\cos x + 1 = 0$

$\cos x = -\frac{1}{4}$

$x = 104.478° \text{ and } 255.522°$

$\text{or } x = 1.823 \text{ rad and } 4.965 \text{ rad}$

6. $5\sin^2 x - 13\sin x - 6 = 0$

$5\sin^2 x - 15\sin x + 2\sin x - 6 = 0$

$5\sin x(\sin x - 3) + 2(\sin x - 3) = 0$

$(5\sin x + 2)(\sin x - 3) = 0$

$\sin x = -\frac{2}{5}$ ~~$\sin x = 3$~~

$x = 203.58° \text{ and } 336.42°$

$\text{or } x = 3.553 \text{ rad and } 5.872 \text{ rad}$

7. $6\cos^2 x + \cos x - 2 = 0$

$6\cos^2 x + 4\cos x - 3\cos x - 2 = 0$

$2\cos x(3\cos x + 2) - (3\cos x + 2) = 0$

$(2\cos x - 1)(3\cos x + 2) = 0$

$\cos x = -\frac{1}{2}$ $\quad \cos x = -\frac{2}{3}$

$x = 60° \text{ and } 300°$ $\quad x = 131.8° \text{ and } 228.2°$

$\text{or } x = \frac{\pi}{3} \text{ and } \frac{5\pi}{3}$ $\quad x = 3.98 \text{ rad and } 5.44 \text{ rad}$

III. Practice – Use properties of trig functions to help you solve these.

Find all solutions of x in $[0,2\pi)$. You may express your answers in degrees or radians.

8. $2\sin^2 x + 3\cos x - 3 = 0$

$2(1-\cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$-2\cos^2 x + 3\cos x - 1 = 0$

$-2\cos^2 x + 2\cos x + \cos x - 1 = 0$

$-2\cos x(\cos x - 1) + (\cos x - 1) = 0$

$(-2\cos x + 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = 1$

$x = 60°, 300°$ $\quad$ $x = 0\ rad$

$or\ x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ $or\ x = 0\ rad$

9. $2\sin x + \cos x = 0$

$2\sin x = -\cos x$

$-2\tan x = 1$

$\tan x = -\frac{1}{2}$

$x = 153.43°\ and\ 333.43°$

$or\ x = 2.68\ rad\ and\ 5.82\ rad$

10. $2\cos^2 x - 3\cos x + 1 = 0$

$2\cos^2 x - 2\cos x - \cos x + 1 = 0$

$2\cos x(\cos x - 1) - (\cos x - 1) = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = 1$

$x = 60°\ and\ 300°$ $\quad$ $x = 0$

$or\ x = \frac{\pi}{3}, \frac{5\pi}{3}$

11. $\cos x + 1 = \sin x$

$(\cos x + 1)^2 = (\sin x)^2$

$\cos^2 x + 2\cos x + 1 = \sin^2 x$

$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$\cos x = 0$ $\quad$ $\cos x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = \pi$