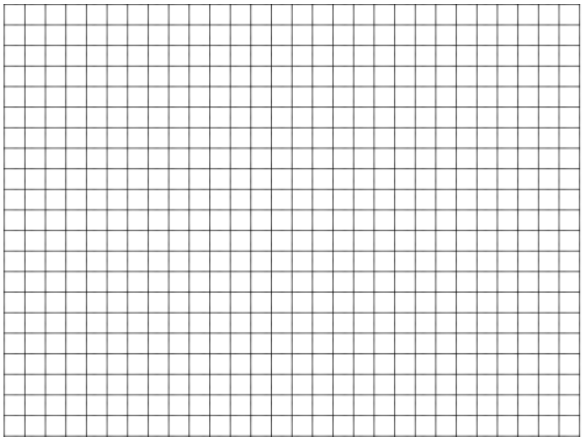
Name: _____Kevin Braza – KEY ___________________________ Date: __________________ IB Math A&A SL

Lesson 6.8 – Solving Trigonometric Equations III – x-intercepts and Modeling

Warm-Up:

1. Consider the function $f(x) = 2\cos x + 1$
   a. Sketch a graph of the function below.



   b. Algebraically find the intercepts from $[0, 2\pi)$.

$y = 2\cos 0 + 1 = 2(1) + 1 = 3$

y-int: (0,3)

$0 = 2\cos x + 1$ $\quad\quad -\frac{1}{2} = \cos x$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

x-int: $\left(\frac{2\pi}{3}, 0\right)$; $\left(\frac{4\pi}{3}, 0\right)$

2. Solve for all values of x, $[0, 2\pi)$. (Express your answers in radians).
   $2\sin x + \cos x = 0$
   $2\sin x = -\cos x$ $\quad\quad \tan x = 0$
   $-2\tan x = 0$ $\quad\quad x = 0, \pi$
3. Mr. Braza is taking an intensive ride on his 5th bike. At $t = 2.3$ seconds the peddle on his bike is closest to the ground at a height of 8 inches. The pedal reaches its highest point of 22 inches 0.7 seconds later.
   a. Find an equation that represents the height of the peddle over time.

Lowest = $8in$, Highest = $22in$, Midline = $15in$. Amplitude: $7\ in$

Is first at lowest at $2.3s$ and highest at $3.0s$. Total period = $1.4s$

$$f(t) = 7\cos\left(\frac{2\pi}{1.4}(t - 3)\right) + 15$$

   b. Find all the times that the pedal is exactly one foot off the ground in the first 5 seconds of his ride.

$12 = 7\cos\left(\frac{2\pi}{1.4}(t - 3)\right) + 15$

$-3 = 7\cos\left(\frac{2\pi}{1.4}(t - 3)\right)$

$-\frac{3}{7} = \cos\left(\frac{2\pi}{1.4}(t - 3)\right)$

$\cos^{-1}(-\frac{3}{7}) = \frac{2\pi}{1.4}(t - 3)$

$t = 3 + \frac{1.4}{2\pi}\cos^{-1}(-\frac{3}{7})$

$t = 3 + \frac{1.4}{2\pi}(\pi - \cos^{-1}(\frac{3}{7}))$ OR $t = 3 + \frac{1.4}{2\pi}(\pi + \cos^{-1}(\frac{3}{7}))$

$t = 3 + \frac{1.4}{2\pi}(2.01371 + 2\pi n)$ OR $t = 3 + \frac{1.4}{2\pi}(4.26948 + 2\pi n)$

Substituting in values of n, all values of t where $0 < t < 5$.

$t = 0.649,\ 1.151,\ 2.049,\ 2.551,\ 3.449,\ 3.951,\ 4.849$

4. A sharpshooter intends to hit a target at a distance of 1000 yards with a gun that has a muzzle velocity v_0 of 1200 feet per second (see figure). Neglecting air resistance, determine the gun's minimum angle of elevation θ is the range is given by $r = \frac{1}{32} v_0^2 \sin^2 2\theta$.

$3000 = \frac{1}{32}(1200)^2 \sin^2 2\theta$

$\frac{1}{15} = \sin^2 2\theta$

$\theta = \frac{1}{2}\sin^{-1}\sqrt{\frac{1}{15}} =$

$\theta = 7.4816$

5. The monthly sales $S(t)$ (in thousands of units) of a seasonal product are approximated by

$$S(t) = 74.50 + 43.75\sin\left(\frac{\pi t}{6}\right)$$

Where t is the time (in months), with $t = 1$ corresponding to January. Determine the months when sales exceed 100,000.

$100 = 74.50 + 43.75\sin\left(\frac{\pi t}{6}\right)$

$\arcsin\left(\frac{100 - 74.50}{43.75}\right) = \frac{\pi t}{6}$

$\frac{6}{\pi}\arcsin\left(\frac{100-74.50}{43.75}\right) = 1.1884$ months