

Lesson 6.3 – Simplifying Trigonometric Identities

I. Warm-Up

- Evaluate all of the trigonometric values. What do you notice?
 - $\sin(30^\circ)$ and $\cos(60^\circ)$
 - $\sec(45^\circ)$ and $\csc(45^\circ)$
 - $\tan(60^\circ)$ and $\cot(60^\circ)$
 - $\sin(15^\circ)$ and $\cos(75^\circ)$ (Use calculator)
- Which functions are even/odd?
 $f(x) = \sin x$ $f(x) = \cos x$ $f(x) = \tan x$

Cofunction Identities

$$\begin{array}{ll} \sin\left(\frac{\pi}{2} - u\right) = \cos u & \cos\left(\frac{\pi}{2} - u\right) = \sin u \\ \tan\left(\frac{\pi}{2} - u\right) = \cot u & \cot\left(\frac{\pi}{2} - u\right) = \tan u \\ \sec\left(\frac{\pi}{2} - u\right) = \csc u & \csc\left(\frac{\pi}{2} - u\right) = \sec u \end{array}$$

Even/Odd Identities

$$\begin{array}{ll} \sin(-u) = -\sin u & \csc(-u) = -\csc u \\ \cos(-u) = \cos u & \sec(-u) = \sec u \\ \tan(-u) = -\tan u & \cot(-u) = -\cot u \end{array}$$

II. Simplifying More Trig Identities

Prove the following identities are true. Only work on one side of the equation.

- $\cot\left(\frac{\pi}{2} - x\right) \cos(x) = \sin x$
- $\sec\left(\frac{\pi}{2} - x\right) \tan(-x) = -\sec x$
- $\cos^4 x - 2\cos^2 x + 1 = \sin^4 x$
- $(2\csc x + 2)(2\csc x - 2) = 4\cot^2 x$

III. Practice

Prove the following identities are true. Only work on one side of the equation.

$$5. \tan x + \frac{\sec^2 x}{\tan(-x)} = \cot x$$

$$6. \csc\left(\frac{\pi}{2} - x\right) \cot(-x) = -\csc x$$

$$7. \frac{1}{\sec(x)+1} - \frac{1}{\sec(x)-1} = -2 \cot^2 x$$

$$8. \sec^3 x - \sec^2 x - \sec x + 1 = \tan^2 x (\sec x - 1)$$

$$9. \sec^4 x - \tan^4 x = 2 \tan^2 x + 1$$

$$10. \ln(\cos^2 \theta) + \ln(1 + \tan^2 \theta) = 0$$