

Lesson 7.5 – Half-Angle Formulas – Equations and More Proofs

What are the half-angle formulas?

Consider the double-angle formula: $\cos(2\theta) = 1 - 2\sin^2\theta$ and $\cos(2\theta) = 2\cos^2\theta - 1$

Make the substitution $u = 2\theta$ and derive the “Half-angle formula” for sine and cosine.

$$\cos\left(\frac{u}{2}\right)$$

$$\sin\left(\frac{u}{2}\right)$$

1. Determine how you would prove the following identities:

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} \quad \text{OR} \quad \tan\frac{u}{2} = \frac{\sin u}{1+\cos u}$$

Solve for all values of x in the interval $[0, 2\pi)$

2. $2 - \sin^2 x = 2 \cos^2\left(\frac{x}{2}\right)$

3. $\sin\left(\frac{x}{2}\right) + \cos x - 1 = 0$

Use the half-angle formulas to evaluate the following. Use your calculator to verify your answers.

6. $\sin\left(\frac{7\pi}{8}\right)$

7. $\cos(15^\circ)$

8. $\cos(7.5^\circ)$