Name: _____ Date: _____ IB Math A&A SL

Review for Finals (Fall 2022)

Section 1 – No Calculators (5-6 questions ~est. 45 min)

"Gleeful" Graphs of Equations/Functions

1. Be sure you can identify the following for each function:
   - Domain and Range
   - x-intercepts and y-intercepts
   - Asymptotes, holes, vertices, axis of symmetry, end behavior, etc.
   - Transformations from a parent function
   - Odd/Even or Neither
   - Graph of the function

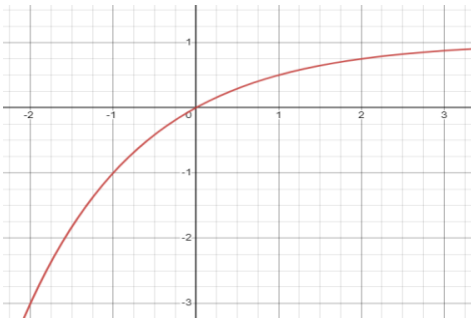
		Easy Example:	Better Example:
a.	Linear Functions: $y = mx + b$	$y = 2x - 1$	$2x + 3y = 5$
b.	Quadratic Functions: $y = ax^2 + bx + c$	$y = x^2 + 6x - 10$	$y = -3x^2 - 12x + 1$
c.	Cubic: $y = a(x - h)^3 + k$	$y = (x + 3)^3 + 1$	$y = -3(2 - x)^3 - 5$
d.	Abs. Value: $y = a\|x - h\| + k$	$y = \|x - 1\| - 2$	$y = -2\|-x + 1\| + 3$
e.	Square Root: $y = a\sqrt{x - h} + k$	$y = 2\sqrt{x - 2} + 3$	$y = \pm\sqrt{-4x - 16} - 2$
f.	Exponential: $y = a^x$ and transformations	$y = 2^x + 1$	$y = -3^{1-x} - 2$
g.	Exponential 'e': $y = e^x$ and transformations	$y = 3e^x - 1$	$y = 2e^{-x+1}$
h.	Logarithmic: $y = \ln x$ and transformations	$y = \ln(x + 1)$	$y = -\ln(-x + 1)$
i.	*Rational Functions	$y = \frac{2x^2+x-3}{-2x^2+3x+9}$	$y = \frac{x^3}{2x^2+4}$

"Fabulous" Functions via Graphs, Charts, and Algebraic Expressions

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	2	-1	0	1	-4	6	4	5	-3

$$g(x) = \begin{cases} 3 - x, & x \le 0 \\ 2^x, & 0 < x < 2 \\ \frac{1}{x}, & 2 \le x \end{cases}$$

$h(x) = graph$



2. Evaluate the following:
   a. $(f - h)(-1) + (fg)(4)$
   b. $g \circ f \circ g(1) + f^{-1} \circ h(-2)$
3. Given $h(x)$ is a continuous exponential function, find an equation for $h(x)$.

"Illuminating" Inverse Functions – Find the inverse function $f^{-1}(x)$ for each.

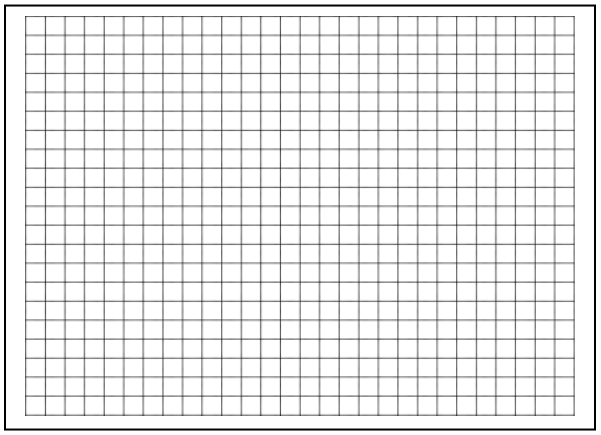
4. $f(x) = \frac{x-2}{1-x}$ $\qquad$ $g(x) = \frac{2^x+1}{2^x-1}$

Graphing "Glamorous" Piecewise Functions

5. Consider the function:

$$f(x) = \begin{cases} |x+2|-1, & x < 0 \\ 3, & x = 0 \\ \sqrt{4-(x-2)^2}, & 0 < x \leq 2 \\ 2-x, & 2 < x \end{cases}$$

a. Graph the function.
b. Identify all intervals where the function is increasing or decreasing.
c. Identify all intervals where the function is continuous.



"Fantastic" Farmer & Fence Problems (Quadratic Optimization/Modeling)

6. Mr. Braza realized that he had to separate the pigs and the cows in his Minecraft farm so he actually had to build two adjacent pastures of equal size with 200 feet of fencing. What dimensions should he use so that the enclosed area will be a maximum? (There is no river in this version of the problem).

Solve for x. Solve for x. Solve for x.

7. $8x^2 + 12x + 5 = 2x^2 + x + 1$
8. $2e^{2x} + e^x = 15$
9. $24 - 2(9^{2x+1}) = -30$
10. $2\sqrt{x} + 3 = x - 5$
11. $\frac{x}{x+1} = \frac{2}{x+1} + 2$
12. $(\log_3 x)(\log_5 3) = 3$
13. $\log(x+3) + \log x = 1$
14. $\log_3(x-1) - \log_3(x+2) = 2$
15. $\sqrt{8-x} = 1 + \sqrt{x+5}$
16. $\sqrt{1-\sqrt{1-\sqrt{x}}} = 1$

Find my ~~iPhone~~ Equations

17. Find the equation of a parabola that has its vertex at $(-3,1)$ and passes through the point $(-1,3)$. Define this as $f(x)$.
18. Find the equation of a line that crosses through the points $(-5,-1)$ and $(1,5)$. Define this as $g(x)$.
19. Find the intersections between the graphs of $f(x)$ and $g(x)$.

"Phenomenal" Proofs

20. Prove that the sum of three consecutive positive integers is divisible by 3.
21. The product of three consecutive integers is increased by the middle integer. Prove that the result is a perfect cube.

"Daring" Derivations (You all knew this was coming)

22. Derive the quadratic formula given a general equation $ax^2 + bx + c = 0$

Section 2 – Calculators Needed (5-6 questions ~est. 45 min)

"Cheerful" Calculator-Required Problems

23. Let $f(x) = x^4 - x^3 - 5x^2 + 3x + 2$, for $x \in \mathbb{R}$.
    (a) Solve the inequality $f(x) < 0$.
    (b) For the graph of $y = f(x)$, find the coordinates of the local maximum point. Round your answers to **three significant figures.**
    (c) The domain of f is now restricted to $[a, b]$ where $a, b \in \mathbb{R}$.
        i. Write down the smallest value of $a < 0$ and the largest value of $b > 0$ for which f has an inverse. Give your answers correct to **three significant figures.**
        ii. For these values a and b, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, showing clearly the coordinates of the end points of each curve.
        iii. Solve the equation $f^{-1}(x) = -1$

"Effervescent" Exponential & Logarithmic Modeling

24. A population of endangered snow leopards, P, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in years. After one year it is estimated that $\frac{P_t}{P_0} = 0.93$.
    a. Find the value of k.
    b. Interpret the meaning of the value of k.
    c. Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.50$.

"**Fantabulous" Financial Math Applications**

25. An eighth-grade student wants to save $40,000 over 5 years to pay for college tuition. The student deposits $20,000 won from science fairs into a savings account that has an interest rate of 6% per annum compounded **monthly** for 5 years.
    a. Show that the student will not be able to reach the target amount.
    b. Find the minimum amount, to the nearest dollar, that the student would need to deposit initially to reach the target amount.

*****Topics I need to cover with you, but will be cut from this semester's final if we don't get there in time*****

"Radiant" Rational Functions

26. Let $f(x) = \frac{9-12x}{cx-20}$, for $x \neq \frac{20}{c}$, where $c \neq 0$.
    a. The line $x = 5$ is a vertical asymptote to the graph of $y = f(x)$.
        i. Find the value of c.
        ii. Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.
    b. The line $y = h$, where $h \in \mathbb{R}$, intersects the graph of $y = |f(x)|$ at exactly one point. Find the possible values of h.

"Serendipitous" Sequences and Series

27. An arithmetic sequence is given by 3, 5, 7, … Write down the value of the common difference, d. Find u_{10}, S_{10}, and given that $u_n = 253$, find the value of n.