Name: ____ Date: ____________ Math 9/10 Honors

Lesson 0.2 – Square Roots, Radicals, and Surds

> Any number written under the **radical sign** $\sqrt{\ }$ is called a **radical.**
>
> The **square root of a**, written as $\sqrt{a}$, is the positive number with the property that $\sqrt{a} \times \sqrt{a} = a$
>
> A **cube root of a** has the property that $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$
>
> And so forth for higher roots.
>
> A real, irrational radical which cannot be simplified to a fraction $\frac{p}{q}$ where p and q are non-radical integers is called a **surd.**
>
> **Note that:**
>
> $$\sqrt{\boldsymbol{ab}} = \sqrt{\boldsymbol{a}} \times \sqrt{\boldsymbol{b}}$$
>
> $$\sqrt{\frac{\boldsymbol{a}}{\boldsymbol{b}}} = \frac{\sqrt{\boldsymbol{a}}}{\sqrt{\boldsymbol{b}}}$$
>
> for $a > 0$ and $b > 0$

I. Introducing Radicals

1. Simplify or rewrite the following as a single surd.

a. $\left(\sqrt{5}\right)^2$

b. $\left(\frac{1}{\sqrt{5}}\right)^2$

c. $(2\sqrt{5})^3$

d. $-2\sqrt{5} \times 3\sqrt{5}$

e. $\sqrt{2} \times \sqrt{3}$

f. $2\sqrt{5} \times 3\sqrt{2}$

g. $\frac{\sqrt{18}}{\sqrt{6}}$

h. $\frac{\sqrt{12}}{2\sqrt{3}}$

2. A radical is in **simplest form** when the number under the radical sign is the smallest possible integer. Write the following radicals in simplest form.

(a) $\sqrt{28}$

(b) $\sqrt{75}$

(c) $\sqrt{36}$

(d) $\sqrt{80}$

(e) $\sqrt{8}$

(f) $\sqrt{32}$

(g) $\sqrt{125}$

(h) $\sqrt{175}$

(i) $\sqrt{45}$

(j) $\sqrt{20}$

(k) $\sqrt{30}$

(l) $\sqrt{18}$

(m) $\sqrt{50}$

(n) $\sqrt{150}$

(o) $\frac{9-\sqrt{75}}{6}$

(p) $\frac{4-\sqrt{8}}{2}$

(q) $\frac{6-\sqrt{12}}{2}$

(r) $\frac{14-\sqrt{60}}{8}$

(s) $\frac{1}{10}(5-\sqrt{200})$

II. Adding and Subtracting Radicals

3. We can add or subtract radicals by simplifying each term then combining like terms.

(a) $3\sqrt{2} + 2\sqrt{2}$

(b) $3\sqrt{2} - 4\sqrt{2}$

(c) $2\sqrt{2} + \sqrt{5} - 6\sqrt{2}$

(d) $5\sqrt{3} + 2\sqrt{5} - \sqrt{3} + 7\sqrt{5}$

(e) $4\sqrt{3} - \sqrt{27}$

(f) $2\sqrt{6} - 2\sqrt{24}$

(g) $-\sqrt{12} + 3\sqrt{3}$

(h) $3\sqrt{8} - 3\sqrt{2}$

(i) $-2\sqrt{3} - 3\sqrt{27}$

(j) $2\sqrt{6} - 3\sqrt{54}$

(k) $-3\sqrt{6} + 3\sqrt{6}$

(l) $\frac{\sqrt{7}}{2} + \frac{\sqrt{7}}{3}$

(m) $\frac{5\sqrt{3}}{6} + \frac{\sqrt{3}}{8}$

(n) $\frac{2\sqrt{11}}{9} + \frac{8\sqrt{11}}{15}$

(o) $\frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{3} + \frac{\sqrt{10}}{4}$

(p) $\sqrt{2} + \frac{5\sqrt{2}}{14} + \frac{7\sqrt{2}}{14}$

III. Multiplying Radicals

4. Simplify the following expressions.

(a) $\sqrt{6} \cdot 4\sqrt{6}$

(b) $-\sqrt{2} \cdot \sqrt{3}$

(c) $\sqrt{12} \cdot \sqrt{15}$

(d) $-3\sqrt{5} \cdot \sqrt{20}$

(e) $\sqrt{9} \cdot \sqrt{3}$

(f) $4\sqrt{8} \cdot \sqrt{2}$

(g) $\sqrt{5} \cdot -2\sqrt{5}$

(h) $\sqrt{15} \cdot 3\sqrt{5}$

(i) $-4\sqrt{8} \cdot \sqrt{10}$

(j) $\sqrt{5}(6 - \sqrt{5})$

(k) $(6 + \sqrt{3})(1 + 2\sqrt{3})$

(l) $(5 - \sqrt{2})^2$

(m) $(7 + 2\sqrt{5})(7 - 2\sqrt{5})$

(n) $\sqrt{5}(3\sqrt{5} - 4\sqrt{3})$

Multiplying Radicals

Multiply numbers that are BOTH OUTSIDE the radical.

Multiply numbers that are BOTH INSIDE the radical.

$2 \cdot 5 =$

$2 \cdot \sqrt{5} =$

$\sqrt{2} \cdot 5 =$

$2\sqrt{3} \cdot 5 =$

$2\sqrt{3} \cdot \sqrt{5} =$

$2\sqrt{3} \cdot 4\sqrt{5} =$

IV. Dividing Radicals and Rationalizing the Denominator

5. Rewrite the following radical expressions in simplest form.

(a) $\frac{\sqrt{8}}{\sqrt{7}}$

(e) $\frac{\sqrt{2}}{\sqrt{6}}$

(b) $\frac{\sqrt{3}}{6\sqrt{7}}$

(f) $\frac{\sqrt{15}}{3\sqrt{6}}$

(c) $\frac{7}{8\sqrt{7}}$

(g) $\frac{\sqrt{21}}{\sqrt{15}}$

(d) $\frac{\sqrt{5}}{\sqrt{3}}$

(h) $\frac{\sqrt{8}}{2\sqrt{7}}$

(e) $\frac{5}{3-\sqrt{2}}$

(f) $\frac{5-\sqrt{2}}{6-\sqrt{2}}$

(g) $\frac{1}{3+5\sqrt{2}}$

(h) $\frac{\sqrt{3}}{1-\sqrt{3}} - \frac{1-2\sqrt{3}}{1+\sqrt{3}}$

(i) $\frac{2+\sqrt{5}}{2-\sqrt{5}} - \frac{\sqrt{5}}{2+\sqrt{5}}$

(j) $\frac{4-\sqrt{3}}{3-2\sqrt{2}} - \frac{2\sqrt{3}}{3+2\sqrt{2}}$

Dividing Radicals

$\frac{6}{2} =$

$\frac{\sqrt{6}}{\sqrt{2}} =$

$\frac{\sqrt{6}}{2} =$

$\frac{12\sqrt{6}}{2} =$

$\frac{12\sqrt{6}}{\sqrt{2}} =$

Simplest form for fractions with radicals:

1. No perfect square under $\sqrt{\quad}$
2. No fractions under $\sqrt{\quad}$
3. No $\sqrt{\quad}$ in a denominator