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Lesson 0.2 – Square Roots, Radicals, and Surds

Any number written under the **radical sign** $\sqrt{}$ is called a **radical.**

The **square root of a**, written as $\sqrt{a}$, is the positive number with the property that $\sqrt{a} \times \sqrt{a} = a$

A **cube root of a** has the property that $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$

And so forth for higher roots.

A real, irrational radical which cannot be simplified to a fraction $\frac{p}{q}$ where p and q are non-radical integers is called a **surd.**

Note that:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

for $a > 0$ and $b > 0$

I. Introducing Radicals

1. Simplify or rewrite the following as a single surd.

a. $\left(\sqrt{5}\right)^2$
$= 5$

b. $\left(\frac{1}{\sqrt{5}}\right)^2$
$= \frac{1}{5}$

c. $(2\sqrt{5})^3$
$= 40\sqrt{5}$

d. $-2\sqrt{5} \times 3\sqrt{5}$
$= -30$

e. $\sqrt{2} \times \sqrt{3}$
$= \sqrt{6}$

f. $2\sqrt{5} \times 3\sqrt{2}$
$= 6\sqrt{10}$

g. $\frac{\sqrt{18}}{\sqrt{6}}$
$= \sqrt{3}$

h. $\frac{\sqrt{12}}{2\sqrt{3}}$
$= 1$

2. A radical is in **simplest form** when the number under the radical sign is the smallest possible integer. Write the following radicals in simplest form.

(a) $\sqrt{28}$
$= 2\sqrt{7}$

(b) $\sqrt{75}$
$= 5\sqrt{3}$

(c) $\sqrt{36}$
$= 6$

(d) $\sqrt{80}$
$= 4\sqrt{5}$

(e) $\sqrt{8}$
$= 2\sqrt{2}$

(f) $\sqrt{32}$
$= 4\sqrt{2}$

(g) $\sqrt{125}$
$= 5\sqrt{5}$

(h) $\sqrt{175}$
$= 5\sqrt{7}$

(i) $\sqrt{45}$
$= 3\sqrt{5}$

(j) $\sqrt{20}$
$= 2\sqrt{5}$

(k) $\sqrt{30}$
$= \sqrt{30}$

(l) $\sqrt{18}$
$= 3\sqrt{2}$

(m) $\sqrt{50}$
$= 5\sqrt{2}$

(n) $\sqrt{150}$
$= 5\sqrt{6}$

(o) $\frac{9-\sqrt{75}}{6}$
$\frac{1}{6}(9 - 5\sqrt{3})$

(p) $\frac{4-\sqrt{8}}{2}$
$\frac{1}{2}(4 - \sqrt{8})$

(q) $\frac{6-\sqrt{12}}{2}$
$\frac{1}{2}(6 - \sqrt{12})$

(r) $\frac{14-\sqrt{60}}{8}$
$\frac{1}{8}(14 - 2\sqrt{15})$

(s) $\frac{1}{10}(5 - \sqrt{200})$
$\frac{1}{10}(5 - 10\sqrt{2})$

II. Adding and Subtracting Radicals

3. We can add or subtract radicals by simplifying each term then combining like terms.

(a) $3\sqrt{2} + 2\sqrt{2}$

$= 5\sqrt{2}$

(b) $3\sqrt{2} - 4\sqrt{2}$

$= -\sqrt{2}$

(c) $2\sqrt{2} + \sqrt{5} - 6\sqrt{2}$

$= -4\sqrt{2} + \sqrt{5}$

(d) $5\sqrt{3} + 2\sqrt{5} - \sqrt{3} + 7\sqrt{5}$

$= 4\sqrt{3} + 9\sqrt{5}$

(e) $4\sqrt{3} - \sqrt{27}$

$= \sqrt{3}$

(f) $2\sqrt{6} - 2\sqrt{24}$

$= -\sqrt{6}$

(g) $-\sqrt{12} + 3\sqrt{3}$

$= \sqrt{3}$

(h) $3\sqrt{8} - 3\sqrt{2}$

$= 3\sqrt{2}$

(i) $-2\sqrt{3} - 3\sqrt{27}$

$= 11\sqrt{3}$

(j) $2\sqrt{6} - 3\sqrt{54}$

$= -11\sqrt{6}$

(k) $-3\sqrt{6} + 3\sqrt{6}$

$= 0$

(l) $\frac{\sqrt{7}}{2} + \frac{\sqrt{7}}{3}$

$\frac{5\sqrt{7}}{6}$

(m) $\frac{5\sqrt{3}}{6} + \frac{\sqrt{3}}{8}$

$\frac{23\sqrt{3}}{24}$

(n) $\frac{2\sqrt{11}}{9} + \frac{8\sqrt{11}}{15}$

$\frac{34\sqrt{11}}{45}$

(o) $\frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{3} + \frac{\sqrt{10}}{4}$

$\frac{13\sqrt{10}}{12}$

(p) $\sqrt{2} + \frac{5\sqrt{2}}{14} + \frac{7\sqrt{2}}{14}$

$\frac{13\sqrt{2}}{7}$

III. Multiplying Radicals

4. Simplify the following expressions.

(a) $\sqrt{6} \cdot 4\sqrt{6}$

$= 24$

(b) $-\sqrt{2} \cdot \sqrt{3}$

$= -\sqrt{6}$

(c) $\sqrt{12} \cdot \sqrt{15}$

$= 6\sqrt{5}$

(d) $-3\sqrt{5} \cdot \sqrt{20}$

$= -30$

(e) $\sqrt{9} \cdot \sqrt{3}$

$= 3\sqrt{3}$

(f) $4\sqrt{8} \cdot \sqrt{2}$

$= 16$

(g) $\sqrt{5} \cdot -2\sqrt{5}$

$= -10$

(h) $\sqrt{15} \cdot 3\sqrt{5}$

$= 15\sqrt{3}$

(i) $-4\sqrt{8} \cdot \sqrt{10}$

$= -16\sqrt{5}$

(j) $\sqrt{5}(6 - \sqrt{5})$

$= 6\sqrt{5} - 5$

(k) $(6 + \sqrt{3})(1 + 2\sqrt{3})$

$= 12 + 13\sqrt{3}$

(l) $(5 - \sqrt{2})^2$

$27 - 10\sqrt{2}$

(m) $(7 + 2\sqrt{5})(7 - 2\sqrt{5})$

$= 29$

(n) $\sqrt{5}(3\sqrt{5} - 4\sqrt{3})$

$-4\sqrt{15} + 15$

Multiplying Radicals

Multiply numbers that are BOTH OUTSIDE the radical.

Multiply numbers that are BOTH INSIDE the radical.

$2 \cdot 5 = 10$

$2 \cdot \sqrt{5} = 2\sqrt{5}$

$\sqrt{2} \cdot 5 = 5\sqrt{2}$

$2\sqrt{3} \cdot 5 = 10\sqrt{3}$

$2\sqrt{3} \cdot \sqrt{5} = 2\sqrt{15}$

$2\sqrt{3} \cdot 4\sqrt{5} = 8\sqrt{15}$

IV. Dividing Radicals and Rationalizing the Denominator

5. Rewrite the following radical expressions in simplest form.

(a) $\frac{\sqrt{8}}{\sqrt{7}} = \frac{\sqrt{56}}{7}$

(e) $\frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{3}}{3}$

(b) $\frac{\sqrt{3}}{6\sqrt{7}} = \frac{\sqrt{21}}{42}$

(f) $\frac{\sqrt{15}}{3\sqrt{6}} = \frac{\sqrt{10}}{6}$

(c) $\frac{7}{8\sqrt{7}} = \frac{\sqrt{7}}{8}$

(g) $\frac{\sqrt{21}}{\sqrt{15}} = \frac{\sqrt{35}}{5}$

(d) $\frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$

(h) $\frac{\sqrt{8}}{2\sqrt{7}} = \frac{4\sqrt{7}}{7}$

(e) $\frac{5}{3-\sqrt{2}} = \frac{15+5\sqrt{2}}{7}$

(f) $\frac{5-\sqrt{2}}{6-\sqrt{2}} = \frac{28-\sqrt{2}}{34}$

(g) $\frac{1}{3+5\sqrt{2}} = \frac{5\sqrt{2}-3}{41}$

(h) $\frac{\sqrt{3}}{1-\sqrt{3}} - \frac{1-2\sqrt{3}}{1+\sqrt{3}} = 2 - 2\sqrt{3}$

(i) $\frac{2+\sqrt{5}}{2-\sqrt{5}} - \frac{\sqrt{5}}{2+\sqrt{5}} = -14 - 2\sqrt{5}$

(j) $\frac{4-\sqrt{3}}{3-2\sqrt{2}} - \frac{2\sqrt{3}}{3+2\sqrt{2}} = 12 + 8\sqrt{2} - 9\sqrt{3} + 2\sqrt{6}$

Dividing Radicals

$\frac{6}{2} = 3$

$\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$

$\frac{\sqrt{6}}{2} = \frac{\sqrt{6}}{2}$

$\frac{12\sqrt{6}}{2} = 6\sqrt{6}$

$\frac{12\sqrt{6}}{\sqrt{2}} = 12\sqrt{3}$

Simplest form for fractions with radicals:

1. No perfect square under $\sqrt{\ }$
2. No fractions under $\sqrt{\ }$
3. No $\sqrt{\ }$ in a denominator