

Lesson 0.3 – Introduction to Variables

Some quantities in life will vary with time and place, like the temperature outside, your age and your height, etc. In algebra, a **variable** is a symbol (usually a letter) that stands in for an unknown value. A variable is a quantity that can be changed and is not fixed. We usually use x and y as our variables, but we can use any letter. A number in front of a variable is called a **coefficient** and a number by itself is called a **constant**.

I. Evaluating Variable Expressions at a Given Value

1. Replace the variable with a number and evaluate:
 - (a) $3x$, when $x = 2$. _____
 - (b) $5y$, when $y = 7$. _____
 - (c) $2z + 1$, when $z = 4$. _____
 - (d) $2x - 3y$, when $x = 5$ and $y = 2$. _____
 - (e) $4y + 2z$, when $y = 3$ and $z = 5$. _____
 - (f) $3x^2 + 2x - 1$, when $x = 4$. _____
 - (g) $3x + 2y$, when $x = 2$ and $y = 5$. _____
 - (h) $4x - 3y$, when $x = 3$ and $y = 2$. _____
 - (i) $5y^2 + 3y + 1$, when $y = 2$. _____

II. Modeling Situations with Variables

2. Write an equation for the given situation using a variable:
 - (a) The sum of some number n and eight. _____
 - (b) Four times some number x . _____
 - (c) Five less than a number m . _____
 - (d) The cost of a shirt is \$20. If you buy n shirts, how much will you spend in total? _____
 - (e) A car travels at a speed of 60 miles per hour. If it travels for t hours, how far will it go? _____
 - (f) A rectangle has a length of 5 meters and a width of w meters. What is its area in terms of w ? _____
 - (g) If the price of a book is \$15, and you buy x number of books, what will be the total cost? _____

III. Combining Like Terms – $2 \text{ apples} + 3 \text{ apples} = 5 \text{ apples}$. $2x + 3x = 5x$

3. Simplify the following expressions by combining like terms:
 - (a) $3x + 4x - 2x$ _____
 - (b) $5y - 2y + 7y - 3y$ _____
 - (c) $2a + 3b + 4a - 5b$ _____
 - (d) $6x^2 + 3x^2 - 2x^2$ _____
 - (e) $4y^3 - 2y^3 + y^3$ _____
 - (f) $2x + 3y - x - 4y$ _____
 - (g) $2a + 3b - 4a + 5b$ _____
 - (h) $5x^2 - 2x^2 + 7x - 3x$ _____
 - (i) $3y^3 - 2y^3 + 5y^2 - 3y^2 + y$ _____
 - (j) $2a^{2b} + 3ab^2 - 4a^{2b} + 5ab^2$ _____
 - (k) $3a^2 - 2b^2 + 5a^2 - 3b^2 + a - b$ _____

4. Simplify the following expressions by distributing and combining like terms.

- | | |
|---|-------|
| (a) $3(2x + 4)$ | _____ |
| (b) $4(3y - 2) - 2(2y + 1)$ | _____ |
| (c) $2x(x + 3) + 3(2x - 1)$ | _____ |
| (d) $3(x + 2) - 4(x - 1)$ | _____ |
| (e) $4(2a + 3b) - 2(3a - b)$ | _____ |
| (f) $2(4x^2 + 3x) - 3(2x^2 - x)$ | _____ |
| (g) $3(x + 2) - 2(x - 3) + 4(2x + 1)$ | _____ |
| (h) $2(a + 3b) - 3(2a - b) + 4(3a + 2b)$ | _____ |
| (i) $3(x + 2y) + 2(x - y)$ | _____ |
| (j) $4(2a - b) - 2(3a + 4b) + 5(2a + 3b)$ | _____ |
| (k) $2(x + 3y) - 3(x - y) + 4(2x + y)$ | _____ |
| (l) $3(4x^2 - 2x) - 2(3x^2 + 4x) + 5(2x^2 - x)$ | _____ |
| (m) $4(3a + 2b) - 2(2a - 3b) + 3(4a - b)$ | _____ |

5. Simplify the following rational expressions using property of exponents.

- | | | | |
|--------------------------------|-------|---------------------------------------|-------|
| (a) $\frac{18x^6}{27x^4}$ | _____ | (g) $\frac{16a^2b^3c^4}{20a^7b^2c^2}$ | _____ |
| (b) $\frac{3x^2}{12x}$ | _____ | (h) $\frac{120x^3y}{25xy^5}$ | _____ |
| (c) $\frac{10a^3b}{-15ab^3}$ | _____ | (i) $\frac{-16x^2y^7}{12x^5y^3z^4}$ | _____ |
| (d) $\frac{36k^3m}{24k^4mn^5}$ | _____ | (j) $\frac{3x^2+6x}{2x}$ | _____ |
| (e) $\frac{12x^2}{9x^2y}$ | _____ | (k) $\frac{5y^3-10y^2+5y}{y^2}$ | _____ |
| (f) $\frac{42x^2}{-36x^3}$ | _____ | (l) $\frac{6x^2+9x}{3x}$ | _____ |

6. Add or subtract the following rational expressions.

- | | | | |
|--|-------|---|-------|
| (a) $\frac{9}{15x} + \frac{2}{15x}$ | _____ | (g) $\frac{y}{y^2-9} + \frac{5}{y^2-9}$ | _____ |
| (b) $\frac{5x}{7} - \frac{2x}{7}$ | _____ | (h) $\frac{8}{2x^2} + \frac{3}{2x^2}$ | _____ |
| (c) $\frac{4x}{2x+3} + \frac{7}{2x+3}$ | _____ | (i) $\frac{2}{x+1} + \frac{1}{x+1}$ | _____ |
| (d) $\frac{2}{5x+9} + \frac{x}{5x+9}$ | _____ | (j) $\frac{x-1}{3x+4} + \frac{2x+9}{3x+4}$ | _____ |
| (e) $\frac{5}{8a} - \frac{2}{8a}$ | _____ | (k) $\frac{5x}{3x^2} - \frac{4}{3x^2}$ | _____ |
| (f) $\frac{7}{x-5} - \frac{4}{x-5}$ | _____ | (l) $\frac{7x+4}{x^2+3x+2} - \frac{3x-2}{x^2+3x+2}$ | _____ |

7. Add or subtract these rational expressions. Show your common denominators.

(a) $\frac{5}{8} - \frac{3}{8x}$ _____

(b) $\frac{2}{4x+12} + \frac{7}{x+3}$ _____

(c) $\frac{5}{4x} + \frac{3}{2x}$ _____

(d) $\frac{5}{x} - \frac{2}{y}$ _____

(e) $\frac{1}{2x} + \frac{1}{3y}$ _____

(f) $\frac{2}{3x} + \frac{1}{3y}$ _____

(g) $\frac{2}{3x} - \frac{3}{4y}$ _____

(h) $\frac{7}{5x} - \frac{6}{7y}$ _____

(i) $\frac{3}{4x} + \frac{1}{6y} - \frac{5}{8x}$ _____

(j) $\frac{2}{3x} - \frac{4}{5y} + \frac{1}{2x}$ _____

(k) $\frac{4}{5x} - \frac{3}{4y} - \frac{2}{3x} + \frac{5}{6x}$ _____

(l) $\frac{1}{2x} + \frac{1}{3y} - \frac{5}{6x} - \frac{1}{4y}$ _____

(m) $\frac{3}{7x} - \frac{5}{12y} + \frac{2}{3x} - \frac{1}{4y}$ _____

(n) $\frac{2}{3x} + \frac{4}{5y} - \frac{7}{8x} - \frac{3}{4y}$ _____

8. Simplify the following expressions with only positive exponents.

(a) $(3a^2)^3$ _____

(j) $(6yx^4)^2$ _____

(b) $(3n^4)^4$ _____

(k) $(u^4v^3)^2$ _____

(c) $(3x^4)^4$ _____

(l) $(2x^4y^4)^4$ _____

(d) $(6b^2)^2$ _____

(m) $(3x^2 \cdot 2x^2)^2$ _____

(e) $(7y^4)^2$ _____

(n) $(2p^3 \cdot 2p)^2$ _____

(f) $(3ab^4)^4$ _____

(o) $(4n^3 \cdot n^2)^2$ _____

(g) $(2x^4y^4)^3$ _____

(p) $(3x \cdot 2x)^2$ _____

(h) $(5mn^3)^3$ _____

(q) $(4x^4 \cdot x^4)^3$ _____

(i) $(x^2y^2)^2$ _____

(r) $(4n^4 \cdot n)^2$ _____

9. Rewrite the following expressions with positive exponents.

(a) a^{-3} _____

(h) $\frac{(-2)^0 r^{-2}}{x^{-3}}$ _____

(b) $x^2 c^{-4}$ _____

(i) $5^{-2} x^2$ _____

(c) $\frac{1}{x^{-3}}$ _____

(j) $(-4)^2 a^{-2}$ _____

(d) $-4x^{-2}$ _____

(k) $\frac{(-2)^{-3} a^4}{b^{-2}}$ _____

(e) $a^{-3} c^0 x^4$ _____

(l) $\frac{x^4 b^{-1}}{3^{-2} a^2}$ _____

(f) $\frac{8c^{-2}}{d}$ _____

(m) $\frac{4^{-2} x^2 c^{-3}}{5^{-2} b^{-2}}$ _____

(g) $\frac{-10a^{-1}}{c^{-2}}$ _____

(n) $\frac{(-2)^{-3} a^{-4} c}{4^{-2} x e^{-3}}$ _____

10. Simplify the following expressions with positive exponents in the final answer.

(a) $\frac{(3a^4 b^2)(6ab^3)}{9a^3 b^4}$ _____

(b) $\frac{(4m^4 n^2)(6m^2 n^4)}{-3m^6 n^6}$ _____

(c) $\frac{(-9x^3 y^6)(8x^7 y^4)}{(2x^3 y^3)(-6x^3 y)}$ _____

(d) $\frac{(-7a^2 b^4)(4a^3 b^5)}{(-2ab^2)^3}$ _____

(e) $\frac{(2xy^2)^3(3x^5 y^4)}{4x^2 y^5}$ _____

(f) $\frac{(-3p^2 q^5)(-4pq^3)^2}{8p^4 q^4}$ _____

(g) $\frac{(-5x^4 y^5)^2(2xy^2)^3}{(10x^3 y^8)^2}$ _____

11. Simplify the following monomial radical expressions:

(a) $\sqrt{16x^2}$ _____

(e) $\sqrt{49y^2 z^4}$ _____

(b) $\sqrt{25y^4}$ _____

(f) $\sqrt{16x^3 y^4}$ _____

(c) $\sqrt{4a^4}$ _____

(g) $\sqrt{25a^2 b^4 c^6}$ _____

(d) $\sqrt{36x^3}$ _____

(h) $\sqrt{9x^4 y^6}$ _____