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Lesson 0.4 – Sets and Venn Diagrams (Math 9/10 Book pages 28-46)

I. Theory of Knowledge, Motivating Number Sets

A **set** is any collection of numbers or objects, and each number or object in a set is called an **element** or a **member** of the set. Any defined object exists either in or out of the set.

Often times a capital letter is used to represent a set, and curly braces are used to denote the elements: {}

For example:

- If V is the set of all vowels, then $V = \{vowels\} = \{a, e, i, o, u\}$
- If E is the set of all even numbers, then $E = \{even\ numbers\} = \{2,4,6,8,10,12, \ldots\}$

We use the symbol $\in$ to mean *an element is a set of* and $\notin$ to mean *is not an element of.*

So for the set $E = \{2,4,6,8,10,12, \ldots\}$, we can say $6 \in E$ but $11 \notin E$.

Special Number Sets
****to remember****

$\mathbb{N} = \{0,1,2,3,4,5,6,7 \ldots\}$ is called the set of all **natural numbers.**

$\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, \ldots\}$ is the set of all **integers.**

$\mathbb{Z}^+ = \{+1, +2, +3, +4, \ldots\}$ is the set of all **positive integers.**

$\mathbb{Q}$ is the set of all **rational numbers,** or numbers which can be written in the form $\frac{p}{q}$ where p and q are integers, $q \neq 0$.

$\mathbb{R}$ is the set of all **real numbers**, all numbers which can be placed on a number line including $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$, etc.

The number of elements in a set S is written as $n(S)$. A set which contains a finite number of elements is called a **finite set,** and a set which contains an infinite number of elements is called an **infinite set.**

For example:

- The set of vowels V has 5 elements, V is a finite set, and $n(V) = 5$.
- The set of even numbers E is an infinite set.

The set { } or $\varnothing$ is called the **empty set** or **null set,** and contains no elements.

1. Write the following statements using set notation:
   a. 8 is an element of set P. $8 \in P$
   b. k is not an element of set S. $k \notin S$
   c. 14 is not an element of the set of all odd numbers. $14 \notin odd$
   d. There are 9 elements in set Y. $9 \in Y$

2. For each of the following sets, list the elements of the set, determine whether the set if finite or infinite, and if the set is finite, find the number of elements in the set.
   a. $A =$ {factors of 6} $\{1,2,3,6\}$ finite, $n(A) = 4$
   b. $B =$ {multiples of 6} $\{6,12,18,24 \ldots\}$, infinite
   c. $C =$ {factors of 17} $\{1,17\}$, finite, $n(C) = 2$
   d. $D =$ {multiples of 17} $\{17,34,51,68, \ldots\}$, infinite
   e. $E =$ {prime numbers less than 20} $\{2,3,5,7,11,13,17,19\}$, finite, $n(E) = 8$
   f. $F =$ {composite numbers between 10 and 30} $\{12,14,15,16,18,20,21,22,24,25,26,27,28\}$ finite, $n(F) = 13$

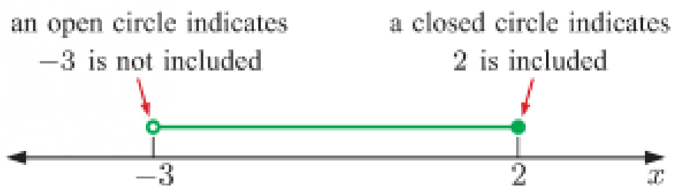
3. Let M_3 be the set of all multiples of 3, and F_{60} be the set of factors of 60.
   a. List the first 8 elements of M_3 in set notation. $\{3,6,9,12,15,18,21,24\}$
   b. List the elements of F_{60} in set notation. $\{1,2,3,4,5,6,10,12,15,20,30,60\}$
   c. What elements are both in M_3 and F_{60}? $\{3,6,12,15\}$

II. Interval Notation

The notation $\{x| \dots\}$ is used to describe "the set of all x such that .....".

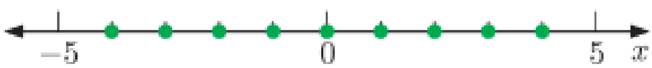
4. Describe the following number lines or statements using interval set builder notation.

(a)



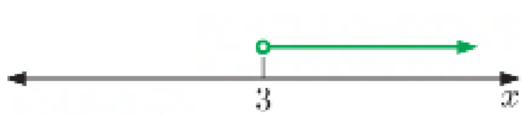
$\{x| -3 < x \leq 2, x \in \mathbb{R}\}$

(b)



$\{x| -5 < x < 5, x \in \mathbb{Z}\}$

(c)



$\{x|x > 3\}$

(d)

$\{x|x \leq -1, x \geq 2\}$

(e) The set of all real numbers greater than 7. $\{x|x > 7\}$

(f) The set of all integers between -8 and 15. $\{x| -8 < x < 15\}$

(g) The set of all rational numbers between 4 and 6, including 4. $\{x|4 \leq x < 6\}$

III. Subsets, Complements & Venn Diagrams

5. Suppose $A = \{1,2,3,4,5,6,7\}, B = \{2,3,5\}$, and $C = \{3,5,8\}$. Decide whether B or C are subsets of A.

Every element of B is in A, but not every element of C is in A, so $B \subseteq A$ but not C.

6. Suppose $U = \{x|x \leq 12, x \in \mathbb{Z}\}$. Find the complement

(a) $A =$ {even numbers in U}

$A' =$ {odd numbers in U}

$= \{1,3,5,7,9,11\}$

(b) $B =$ {prime numbers in U}

$B' = \{1,3,5,7,9,11\}$

Subsets and Complements

Suppose A and B are two sets. A is a **subset** of B if every element of A is also an element of B. We write $A \subseteq B$ to denote a subset relationship between A and B.

The **universal set** U is the set of all elements under consideration. The **complement** of A, called A', is the set of all elements in U which are *not* in A.

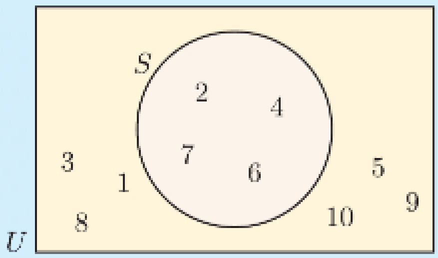
$$A' = \{x|x \notin A, \in U\}$$

7. Consider the set $S = \{2,4,6,7\}$ within the universal set $\{x | x \leq 10, x \in \mathbb{Z}^+\}$.
   (a) Draw a Venn diagram to show S.
   (b) List the elements of the complement set S'
   (c) Find $n(S), n(S'), n(U)$.

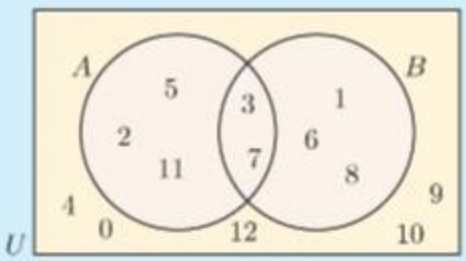
If A and B are two sets, then:

- $A \cap B$ is the **intersection** of A and B, and consists of all elements which are in **both** A and B.
- $A \cup B$ is the **union** of A and B, and consists of all elements which are in A **or** B (or both).

a



8. Consider $U = \{x | 0 \leq x \leq 12, x \in \mathbb{Z}\}$, $A = \{2,3,5,7,11\}$, and $B = \{1,3,6,7,8\}$. Illustrate A and B on a Venn diagram. State the sets $A \cap B$ and $A \cup B$.



3 and 7 are in both A and B, so the circles representing A and B must overlap.

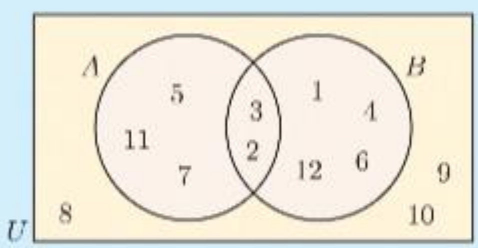
We place 3 and 7 in the overlap, then fill in the rest of A and the rest of B.

The remaining elements of U are placed outside the two circles.

9. Suppose U ={positive integers ≤ 12}, A ={primes ≤ 12}, and B ={factors of 12}.
   (a) List the elements of the sets A and B.
   (b) Show the sets $A, B,$ and U on a Venn diagram.
   (c) List the elements in: i. A' ii. $A \cap B$ iii. $A \cup B$
   (d) Find: i. $n(A \cap B)$ ii. $n(A \cup B)$ iii. $n(B')$

a $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 2, 3, 4, 6, 12\}$

b



c **i** $A' = \{1, 4, 6, 8, 9, 10, 12\}$ **ii** $A \cap B = \{2, 3\}$
iii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 11, 12\}$

d **i** $n(A \cap B) = 2$ **ii** $n(A \cup B) = 9$
iii $B' = \{5, 7, 8, 9, 10, 11\}$, so $n(B') = 6$

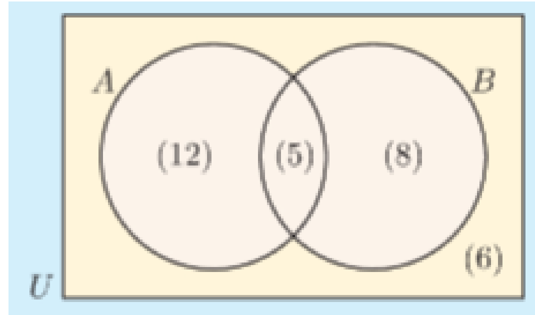
Two sets are **disjoint** or **mutually exclusive** if they have no elements in common.

If A and B are disjoint then $A \cap B = \emptyset$

10. Draw a Venn diagram for Problem #3. Find: $n(M_3 \cap F_{60})$, $n(M_3 \cup F_{60})$, and $n(F_{60}{}')$

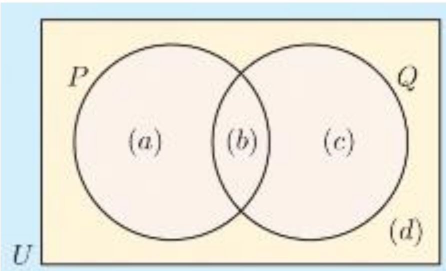
IV. Problem Solving with Venn Diagrams

11. In the Venn diagram given, (5) means that there are 5 elements in the set $A \cap B$.
How many elements are there in:



(a) A $n(A) = 17$
(b) B' $n(B') = 18$
(c) $A \cup B$ $n(A \cup B) = 25$
(d) A, but not B. $n(A\text{, but not } B) = 12$
(e) B, but not A. $n(B\text{, but not } A) = 8$
(f) Neither A nor B? $n(\text{Neither } A \text{ nor } B) = 6$

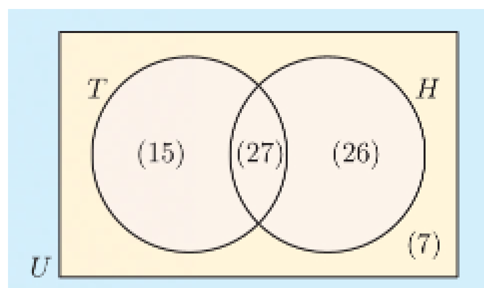
12. Given $n(U) = 25, n(P) = 10, n(Q) = 12$, and $n(P \cap Q) = 3$, find $n(P \cup Q)$ and $n(P$, but not $Q)$.



We see that $b = 3$ {as $n(P \cap Q) = 3$}
$a + b = 10$ {as $n(P) = 10$}
$b + c = 12$ {as $n(Q) = 12$}
$a + b + c + d = 25$ {as $n(U) = 25$}
$\therefore\ b = 3,\ a = 7,$ and $c = 9$
$\therefore\ 7 + 3 + 9 + d = 25$
$\therefore\ d = 6$

a $n(P \cup Q) = a + b + c = 19$ **b** $n(P\text{, but not } Q) = a = 7$

13. The Venn diagram alongside illustrates the number of people in a sporting in sporting club who play tennis (T) and hockey (H).
Determine the number of people:



(a) in the club 75
(b) who play hockey 53
(c) who play both sports 27
(d) who play neither sport 7
(e) who play at least one sport 68