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Lesson 1.2 – Binomial Expansion & Factoring Polynomials (pages 53-64 in Red 9/10 textbook)

**I. Warm-Up:** Expand the following expressions

a. $(x+y)^2$

$x^2+2xy+y^2$

b. $(x+y)^3$

$x^3+3x^2y+3xy^2+y^3$

**II. Binomial Expansion**

1. Expand $(x+y)^4$

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

*What's next?*

**Pascal's Triangle**

1

1 1

1 2 1

1 3 3 1

2. Expand $(x+y)^5$

$x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$

3. Expand $(x-y)^5$

$x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$

**III. Practice on Your Own**

4. Expand $(x-2y)^4$

$x^4-8x^3y+24x^2y^2-32xy^3+16y^4$

5. Expand $(x+2)^5$

$x^5+10y^4+40y^3+80y^2+80y+32$

6. Expand $\left(\frac{x}{2}+\sqrt{y}\right)^3$

$1\left(\frac{x}{2}\right)^3+3\left(\frac{x}{2}\right)^2\sqrt{y}+3\left(\frac{x}{2}\right)\sqrt{y}^2+\sqrt{y}^3$

$\frac{x^3}{8}+\frac{3}{4}x^2\sqrt{y}+\frac{3}{2}xy+\sqrt{y^3}$

## IV. Highest Common Factors & Reordering Expressions

*Factoring is the reverse of expanding, where you rewrite a polynomial expression as a product of its factors.*

Factor all of the following expressions.

7. $6x^2 + 4x = 2x(3x + 2)$

8. $-2x^2 - 4x = -2x(x + 2)$

9. $-4(a + 1) + (a + 2)(a + 1) = (a + 1)(a - 2)$

10. $3ab + d + 3ad + b = (3a + 1)(b + d)$

11. $x^2 + 2x + 5x + 10 = (x + 2)(x + 5)$

12. $x^2 + 3x - 4x - 12 = (x + 3)(x - 4)$

**Recall:**

**FOIL Method**

**First – Outer – Inner – Last**

$(a + b)(c + d) = ac + ad + bc + bd$

**Differences of Squares Pattern**

$(a + b)(a - b) = a^2 - b^2$

**Perfect Square Pattern**

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

## V. Factoring Special Patterns

13. $4 - 9y^2 = (2 + 3y)(2 - 3y)$

14. $9a - 16a^3 = a(3 + 4a)(3 - 4a)$

15. $4x^2 + 4x + 1 = (2x + 1)^2$

16. $8x^2 - 24x + 18 = 2(2x - 3)^2$

**Factoring Quadratic Trinomials**

Consider the expansion of the product

$(x + 2)(x + 5) = x^2 + 5x + 2x + 2 \times 5$

$x^2 + [5x + 2x] + [2 \times 5]$

$x^2 + sum(2 + 5)[x] + product(2 \times 5)$

$x^2 + 7x + 10$

$x^2 + px + q = (x + a)(x + b)$

Where a and b two numbers whose sum is p and whose product is q.

## VI. Factoring Quadratic Trinomials (a = 1)

17. $x^2 - 7x + 12 = (x - 3)(x - 4)$

18. $x^2 - 2x - 15 = (x - 5)(x + 3)$

19. $3x^2 + 6x - 72 = 3(x + 6)(x - 4)$

20. $77 + 4x - x^2 = -(x - 11)(x + 7)$

## VII. Factoring Quadratic Trinomials (a = 1)

21. $3x^2 + 17x + 10 = (3x + 2)(x + 5)$

22. $6x^2 - 11x - 10 = (2x - 5)(3x + 2)$

23. $-5x^2 - 7x + 6 = -(x + 2)(5x - 3)$

## VIII. Practice on Your Own

24. $x^2 - 2x - 15 = (x - 5)(x + 3)$

25. $x^2 - 81 = (x - 9)(x + 9)$

26. $4x^2 - 16 = 4(x - 2)(x + 2)$

27. $16x^2 - 24x + 9 = (4x + 3)(4x + 3)$

28. $x^3 + 27 = (x + 3)(x^2 - 3x + 9)$

29. $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

30. $t^5 + 4t^4 - 5t^3 = t^3(t + 5)(t - 1)$

31. $x^3 + 10x^2 - 36x - 360 = (x + 10)(x - 6)(x + 6)$

32. $4a^2 + 36ab + 81b^2 - 49 =$
$(2a + 9b - 7)(2a + 9b + 7)$

33. $25x^{16} - 9y^6 = (5x^8 - 3y^3)(5x^8 + 3y^3)$

34. $(x - 1)(x + 6)^2 - (x - 1)^2(x + 6) = 7(x - 1)(x + 6)$

35. $343x^{12} - 8y^{15} = (7x^4 - 2x^5)(49x^8 + 14x^4y^5 + 4y^{10})$

**Splitting the Middle Term**

Consider the expansion of the product

$(4x + 3)(x + 2) = 4x^2 + 11x + 6$

Now do it backwards:

$4x^2 + 11x + 6$

$4x^2 + (8x + 3x) + 6$

$(4x^2 + 8x) + (3x + 6)$

$4x(x + 2) + 3(x + 2)$

$(4x + 3)(x + 2)$

p + q = middle term

p * q = last term

**More Special Patterns:**

**Sum of Cubes**

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

**Difference of Cubes**

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$