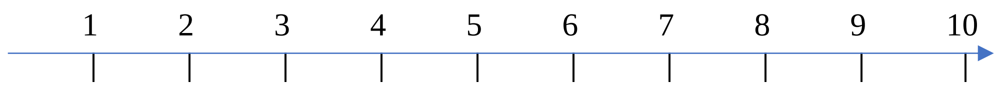
Name: ______________________________ Date: ____________ Math 9/10 Honors

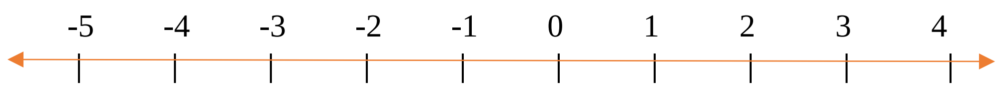
Lesson 1.5 – Imaginary & Complex Numbers

I. Theory of Knowledge, Motivating Complex Numbers

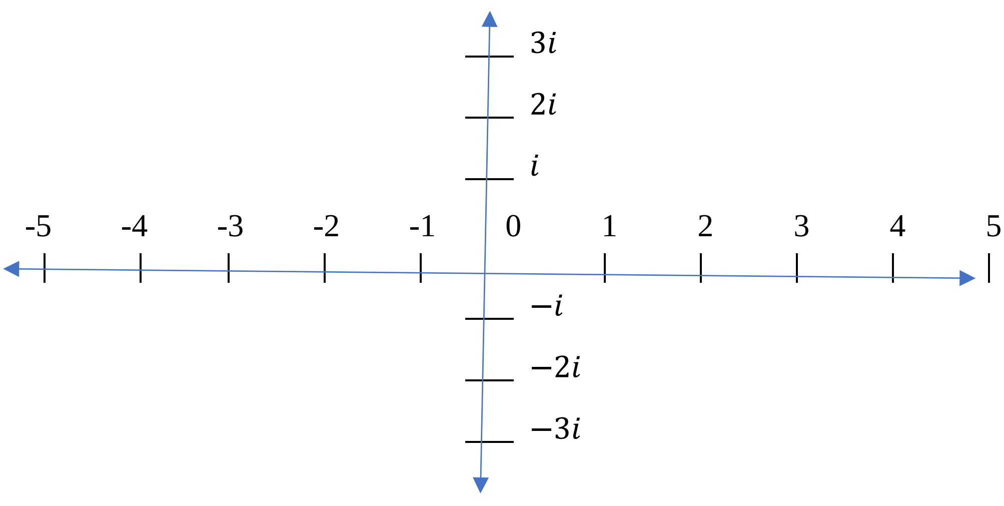
The very first topic you learned in mathematics was probably counting. The set of counting numbers $\{1,2,3,4 \ldots\}$ *are called the* ***natural numbers*** $\mathbb{N}$***,*** *and are represented with the number line.*



At a certain point around 4[th] *grade, we introduced* ***negative numbers****. Negative numbers give you a solution to equations like* $x + 2 = 1$*. From a physical standpoint, we can model situations like debt and reversal of direction using negative numbers. This can be visually represented as a reflection across the number line, and the set of all these numbers is called the* ***real numbers*** $\mathbb{R}$***.***



Here we will introduce the concept of ***imaginary numbers****, which will transform our number line once more by introducing rotations. Let* ***i*** *be defined as some number whose square is* -1*. Imaginary numbers give you a solution to equations like* $x^2 = -1$*.*



A ***complex number*** $\boldsymbol{a + bi}$ *has both a real number part* $\boldsymbol{a}$ *and an imaginary part* $\boldsymbol{bi}$*. We will discuss the practical applications of complex numbers much later, but for now just recognize that they might appear as solutions to some quadratic equations. A complex double-number line like the one drawn above is called an* ***Argand diagram*** *or more commonly a complex coordinate plane.*

Let ***i*** be defined as some number whose square is -1.

Given $\boldsymbol{i = i}$

and $\boldsymbol{i^2 = -1}$

Calculate the following:

$i^3 =$

$i^4 =$

$i^5 =$

$i^6 =$

$i^{59} =$

$i^0 =$

$i^{-1} =$

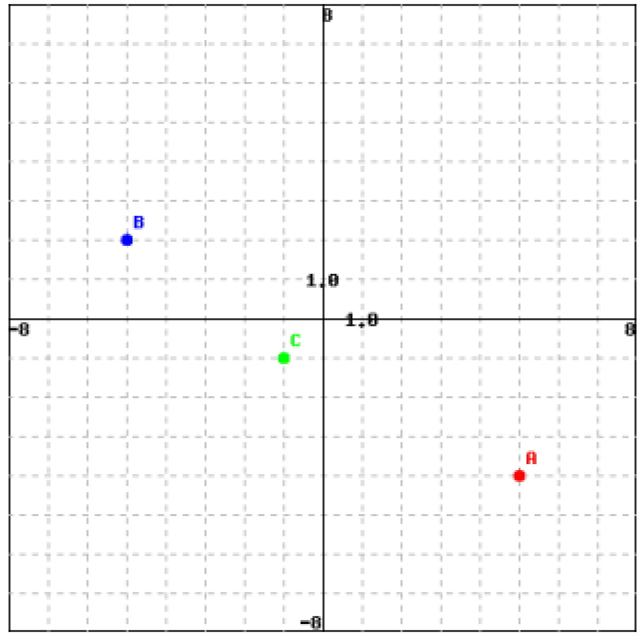
$i^{-2} =$

$i^{-3} =$

$i^{-89} =$

1. Enter the complex coordinates of the following points on the Argand diagram.

   A:
   B
   C:



2. Evaluate the expression $(9 + 4i) + (7 + 6i)$ and write the result in the form $a + bi$.

3. Evaluate the expression $(-5 + 6i) + (-4 + 6i)$ and write the result in the form $a + bi$.

4. Evaluate the expression $(-6 - 4i) - (-8 - 4i)$ and write the result in the form $a + bi$.

5. Evaluate the expression $(-6 + 2i) - (6 + 7i)$ and write the result in the form $a + bi$.

6. Evaluate the expression $(9 - 7i)(-8 - 5i)$ and write the result in the form $a + bi$.

7. Evaluate the expression $(3 + i)(5 + 5i)(-3 + 3i)$ and write the result in the form $a + bi$.

8. Evaluate the expression $((-4-4i)^2-3)i$ and write the result in the form $a+bi$.

9. Evaluate the expression $(a+bi)(a-bi)$ and write the result in the form $a+bi$.

10. Evaluate the expression $(a+bi)^2-(a-bi)^2$ and write the result in the form $a+bi$.

11. Evaluate the expression $(a+bi)^3$ and write the result in the form $a+bi$.

II. Complex Conjugates & Dividing Complex Numbers

12. If $z=4-6i$ then find the values of z^* and $|z|$.

13. Evaluate the expression $\frac{4-6i}{2-5i}$ and write the result in the form $a+bi$.

The **complex conjugate** of a complex number is defined as the number with an equal real part but opposite magnitude imaginary part.

For any complex number $z=a+bi$

Its conjugate is

The **magnitude** of a complex number is its distance from the origin, which from the Pythagorean theorem is

Note that

14. Evaluate the expression $\frac{8-7i}{-8-6i}$ and write the result in the form $a + bi$.

III. Practice on Your Own

15. Evaluate the expression $(i)^2(-5 + i)^2$and write the result in the form $a + bi$.

16. Evaluate the expression $\frac{4-8i}{6-5i}$ and write the result in the form $a + bi$.

17. Evaluate the expression $\frac{-1+3i}{-4-8i}$ and write the result in the form $a + bi$.

18. Evaluate the expression $\frac{-4+4i}{4-6i}$ and write the result in the form $a + bi$.

19. Evaluate the expression $\frac{-6-i}{1+5i}$ and write the result in the form $a + bi$.

20. Evaluate the expression $\frac{-4+3i}{-10+7i}$ and write the result in the form $a + bi$.