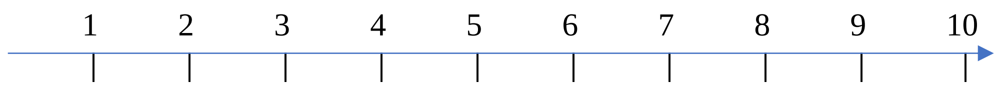
Name: __Kevin Braza - KEY__________________________________ Date: _____________ Math 9/10 Honors

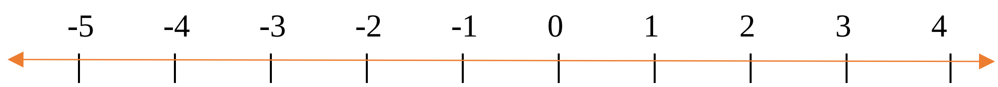
Lesson 1.5 – Imaginary & Complex Numbers

I. Theory of Knowledge, Motivating Complex Numbers

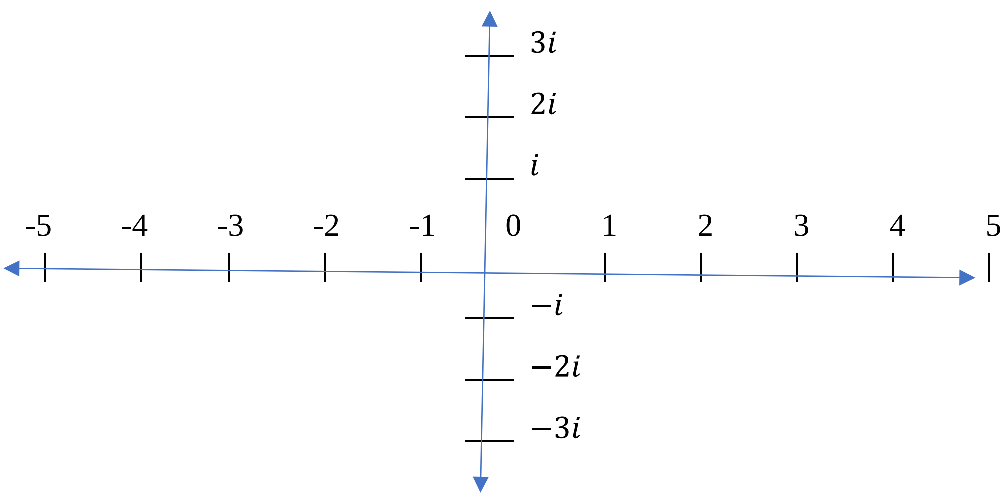
The very first topic you learned in mathematics was probably counting. The set of counting numbers $\{1,2,3,4 \ldots\}$ *are called the* ***natural numbers*** $\mathbb{N}$***,*** *and are represented with the number line.*



At a certain point around 4th grade, we introduced ***negative numbers****. Negative numbers give you a solution to equations like* $x + 2 = 1$. *From a physical standpoint, we can model situations like debt and reversal of direction using negative numbers. This can be visually represented as a reflection across the number line, and the set of all these numbers is called the* ***real numbers*** $\mathbb{R}$***.***



Here we will introduce the concept of ***imaginary numbers****, which will transform our number line once more by introducing rotations. Let* ***i*** *be defined as some number whose square is* -1. *Imaginary numbers give you a solution to equations like* $x^2 = -1$.



A ***complex number*** $\boldsymbol{a + bi}$ *has both a real number part* $\boldsymbol{a}$ *and an imaginary part* $\boldsymbol{bi}$. *We will discuss the practical applications of complex numbers much later, but for now just recognize that they might appear as solutions to some quadratic equations. A complex double-number line like the one drawn above is called an* ***Argand diagram*** *or more commonly a complex coordinate plane.*

Let ***i*** be defined as some number whose square is -1.

Given $\boldsymbol{i = i}$

and $\boldsymbol{i^2 = -1}$

Calculate the following:

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^{59} = -i$

$i^0 = 1$

$i^{-1} = -i$

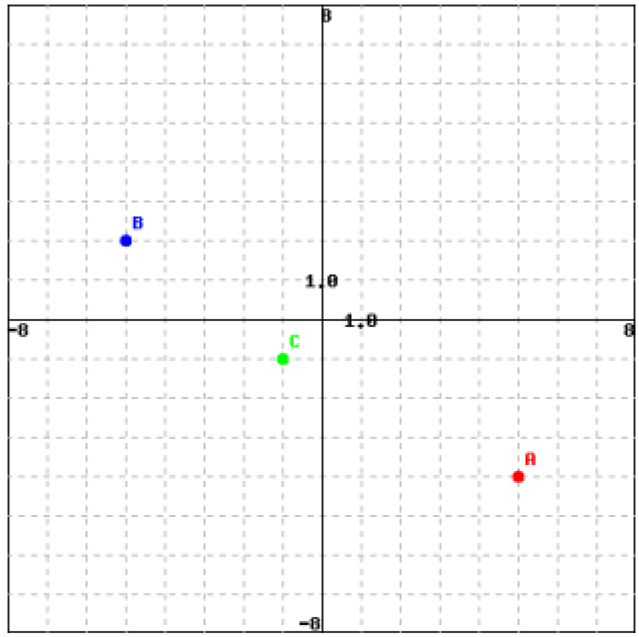
$i^{-2} = -1$

$i^{-3} = i$

$i^{-89} = -i$

1. Enter the complex coordinates of the following points on the Argand diagram.

   A: $5 - 4i$
   B: $5 + 2i$
   C: $-1 - i$



2. Evaluate the expression $(9 + 4i) + (7 + 6i)$ and write the result in the form $a + bi$.

$16 + 10i$

3. Evaluate the expression $(-5 + 6i) + (-4 + 6i)$ and write the result in the form $a + bi$.

$-9 + 12i$

4. Evaluate the expression $(-6 - 4i) - (-8 - 4i)$ and write the result in the form $a + bi$.

2

5. Evaluate the expression $(-6 + 2i) - (6 + 7i)$ and write the result in the form $a + bi$.

$12 - 5i$

6. Evaluate the expression $(9 - 7i)(-8 - 5i)$ and write the result in the form $a + bi$.

$-107 + 11i$

7. Evaluate the expression $(3 + i)(5 + 5i)(-3 + 3i)$ and write the result in the form $a + bi$.

$-90 - 30i$

8. Evaluate the expression $((-4-4i)^2-3)i$ and write the result in the form $a+bi$.

$-32-3i$

9. Evaluate the expression $(a+bi)(a-bi)$ and write the result in the form $a+bi$.

a^2+b^2

10. Evaluate the expression $(a+bi)^2-(a-bi)^2$ and write the result in the form $a+bi$.

$-4abi$

11. Evaluate the expression $(a+bi)^3$ and write the result in the form $a+bi$.

$(a^3-3ab^2)+(3a^2b-b^3)i$

II. Complex Conjugates & Dividing Complex Numbers

12. If $z=4-6i$ then find the values of z^* and $|z|$.

$z^*=4+6i$

$|z|=2\sqrt{13}.$

13. Evaluate the expression $\frac{4-6i}{2-5i}$ and write the result in the form $a+bi$.

$\frac{-2+3i}{5}$

The **complex conjugate** of a complex number is defined as the number with an equal real part but opposite magnitude imaginary part.

For any complex number $z=a+bi$

Its conjugate is $z^*=a+bi.$

The **magnitude** of a complex number is its distance from the origin, which from the Pythagorean theorem is

$$|z|=\sqrt{a^2+b^2}$$

Note that $(z)(z^*)=a^2+b^2=|z|^2$

14. Evaluate the expression $\frac{8-7i}{-8-6i}$ and write the result in the form $a + bi$.

$$\frac{-11 + 52i}{50}$$

III. Practice on Your Own

15. Evaluate the expression $(i)^2(-5 + i)^2$ and write the result in the form $a + bi$.

$-24 + 10i$

16. Evaluate the expression $\frac{4-8i}{6-5i}$ and write the result in the form $a + bi$.

$$-\frac{64 - 28i}{61}$$

17. Evaluate the expression $\frac{-1+3i}{-4-8i}$ and write the result in the form $a + bi$.

$$-\frac{1 - i}{4}$$

18. Evaluate the expression $\frac{-4+4i}{4-6i}$ and write the result in the form $a + bi$.

$$-\frac{10 - 2i}{13}$$

19. Evaluate the expression $\frac{-6-i}{1+5i}$ and write the result in the form $a + bi$.

$$-\frac{11 + 29i}{26}$$

20. Evaluate the expression $\frac{-4+3i}{-10+7i}$ and write the result in the form $a + bi$.

$$\frac{61 - 2i}{149}$$