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Lesson 1.6 – Completing the Square, Quadratic Formula & Discriminant (Red 9/10 Book pages 234-244)

I. Warm-up

1. Solve the equation for x:
   $(x-3)^2 = 16$
   $x - 3 = \pm 4$
   $x = 3 \pm 4$
   $x = 7, x = -1$

Recall: Equations of the form $x^2 = k$

If $x^2 = k$ then
- $x = \pm\sqrt{k},\ if\ k > 0$
- $x = 0,\ if\ k = 0$
- $no\ real\ solutions, if\ k < 0$

2. Solve the equation for x:
   $(x+2)^2 = 11$
   $x + 2 = \pm\sqrt{11}$
   $x = -2 \pm \sqrt{11}$

II. Completing the Square

3. Solve for x by completing the square. Leave your answers in simplest radical form.
   $x^2 + 2x - 2 = 0$
   $x^2 + 2x = 2$
   $x^2 + 2x + 1^2 = 2 + 1^2$
   $(x+1)^2 = 3$
   $x + 1 = \pm\sqrt{3}$

Forming a Perfect Square

Recall that a perfect square binomial has the pattern: $(a+b)^2 = a^2 + 2ab + b^2$

Given $x^2 + 4x - 7 = 0$

$x^2 + 4x = 7$

$x^2 + 4x + 4 = 7 + 4$

$(x+2)^2 = 11$

$x + 2 = \pm\sqrt{11}$

$x = -2 \pm \sqrt{11}$

4. Solve for x by completing the square. Leave your answers in simplest radical form.
   $x^2 - 5x + 3 = 0$
   $x^2 - 5x = -3$
   $x^2 - 5x + \left(\frac{5}{2}\right)^2 - 3 + \left(\frac{5}{2}\right)^2$
   $\left(x - \frac{5}{2}\right)^2 = -3 + \frac{25}{4}$
   $\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$
   $x - \frac{5}{2} = \pm\sqrt{\frac{13}{4}}$
   $x = \frac{5}{2} \pm \sqrt{\frac{13}{4}}$
   $x = \frac{5 \pm \sqrt{13}}{2}$

5. Solve for x by completing the square. Recall that $i^2 = -1$.

$x^2 - 4x + 6 = 0$

$x^2 - 4x = -6$

$x^2 - 4x + 2^2 = -6 + 2^2$

$(x-2)^2 = -2$

$x - 2 = \pm\sqrt{-2}$

$x = 2 \pm \sqrt{-2}$

$x = 2 \pm i\sqrt{2}$

6. Solve for x by completing the square.

$3x^2 + 6x - 2 = 0$

$x^2 + 2x - \frac{2}{3} = 0$

$x^2 + 2x = \frac{2}{3}$

$x^2 + 2x + 1^2 = \frac{2}{3} + 1^2$

$(x+1)^2 = \frac{5}{3}$

$x + 1 = \pm\sqrt{\frac{5}{3}} = \frac{-3\pm\sqrt{15}}{3}$

7. Solve for x by completing the square.

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a}\left(\frac{4a}{4a}\right) + \frac{b^2}{4a^2}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$x + \frac{b}{2a} = \sqrt{\frac{b^2-4ac}{4a^2}}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

III. The Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

8. Solve for x using the quadratic formula:
   $x^2 - 2x - 2 = 0$

   $x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-2)}}{2(1)}$

   $x = \frac{2 \pm \sqrt{4+8}}{2}$

   $x = \frac{2 \pm \sqrt{12}}{2}$

   $x = \frac{2 \pm 2\sqrt{3}}{2}$

   $x = 1 \pm \sqrt{3}$

9. Solve for x using the quadratic formula:
   $2x^2 + 3x - 4 = 0$

   $x = \frac{-3 \pm \sqrt{3^2-4(2)(-4)}}{2(2)}$

   $x = \frac{-3 \pm \sqrt{9+32}}{2(2)}$

   $x = \frac{-3 \pm \sqrt{9+32}}{4}$

   $x = \frac{-3 \pm \sqrt{41}}{4}$

   $x = \frac{-3 \pm \sqrt{41}}{4}$

10. Solve for x using the quadratic formula:
    $(x - 2)(x + 8) = 11$
    $x^2 + 8x - 2x - 16 = 11$
    $x^2 + 6x - 27 = 0$
    $x = \frac{-6 \pm \sqrt{6^2-4(1)(-27)}}{2(1)}$
    $x = \frac{-6 \pm \sqrt{36+108}}{2}$
    $x = \frac{-6 \pm \sqrt{144}}{2}$
    $x = \frac{-6 \pm \sqrt{144}}{2}$

11. Solve for x using the quadratic formula:

$-8x^2 + 6x - 9 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(-8)(-9)}}{2(-8)}$

$x = \frac{-6 \pm \sqrt{36 - 288}}{-16}$

$x = \frac{-6 \pm \sqrt{-252}}{-16}$

$x = \frac{-6 \pm 6i\sqrt{7}}{-16}$

$x = \frac{3 \pm 3i\sqrt{7}}{8}$

The Discriminant

(The value underneath the square root in the quadratic formula will tell the number of real solutions to the quadratic equation.)

$b^2 - 4ac > 0$

2 roots

$b^2 - 4ac = 0$

1 root

$b^2 - 4ac < 0$

0 root

IV. Using the Discriminant

1. For each of the following, use the quadratic formula to find the roots of the equation. What do you notice about the value under the square root?

$x^2 - 2x - 3$ — 2 roots

$x^2 - 2x + 1$ — 1 root

$x^2 - 2x + 3$ — 0 roots

2. Use the discriminant to determine how many zeroes each quadratic will have.

a. $y = x^2 + 3x + 4$

$(3)^2 - 4(1)(4) = 9 - 16 \rightarrow$ negative. Function has no real solutions.

b. $y = -2x^2 + 5x + 1$

3. Find the values of *k* for which the quadratic equation $y = x^2 - 6x + k$.

a. Has two real solutions.

$(-6)^2 - 4(1)(k) > 0$

$36 - 4k > 0$

$36 > 4k$

$k < 9$

b. Has only one real solution.

c. $(-6)^2 - 4(1)(k) = 0$

$36 - 4k = 0$

$36 = 4k$

$k = 9$

d. Has no real solutions (two imaginary solutions).

$(-6)^2 - 4(1)(k) < 0$

$36 - 4k < 0$

$36 < 4k$

$k > 9$