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Lesson 1.7 – Radical Equations & Linear Inequalities (Pages 432-455 in Math 9/10 Textbook)

| I.   Solve Radical Equations | **How to Solve Radical Equations** |

I.   Solve Radical Equations

1. Solve for the only possible solution. Check if your solution is valid or extraneous.

$\sqrt{3x + 9} = 6$

$3x + 9 = 36$

$3x = 27$

$x = 9$

Test: $x = 9$

$\sqrt{3(9) + 9} = 6$

$\sqrt{36} = 6$

2. Solve for the only possible solution. Check if your solution is valid or extraneous.

$\sqrt{-5x - 1} = \sqrt{8x + 5}$

$-5x - 1 = 8x + 5$

$-13x = 6$

$x = -6/13$

Test: $x = -6/13$

$\sqrt{-5(-\frac{6}{13}) - 1} = \sqrt{8(-\frac{6}{13}) + 5}$

$\sqrt{(\frac{30}{13}) - \frac{13}{13}} = \sqrt{(-\frac{48}{13}) + \frac{65}{13}} = \frac{17}{13}$

3. Solve for the only possible solution. Check if your solution is valid or extraneous.

$\sqrt{10 - x} + x = -2$

$10 - x = (-x - 2)^2$

$10 - x = x^2 + 4x + 4$

$x^2 + 5x - 6 = 0$

$(x + 6)(x - 1) = 0$

$x = -6, \cancel{x = 1}$

Test $x = -6$

$\sqrt{10 + 6} - 6 = -2$

Test $x = 1$

$\cancel{\sqrt{10 - 1} + 1 = -2}$

4. Solve the following equation: Check if your solution is valid or extraneous.

$x + \sqrt{2x + 1} = 7$

$\sqrt{2x + 1} = 7 - x$

$2x + 1 = (7 - x)^2$

$2x + 1 = 49 - 14x + x^2$

$0 = 48 - 16x + x^2$

$0 = (x - 12)(x - 4)$

$x = 12, \ x = 4$

Test: $x = 12$

$\cancel{\sqrt{2(12) + 1} = 7 - (12)}$

Test: $x = 4$

$\sqrt{2(4) + 1} = 7 - (4)$

5. Solve the following equation. Check if your solution is valid or extraneous.

$x - 3\sqrt{x} - 10 = 0$

$(\sqrt{x} - 5)(\sqrt{x} + 2) = 0$

$\sqrt{x} = 5, \qquad , x = 25 \qquad \cancel{\sqrt{x} = -2}$

6. Solve the following equation. Check if your solution is valid or extraneous.

$$\sqrt{11 - x^2} - \frac{2}{\sqrt{11 - x^2}} = 1$$

$u - \frac{2}{u} = 1$

$u^2 - 2 = u$

$u^2 - u - 2 = 0$

$(u - 2)(u + 1) = 0$

$u = 2, \ u = -1$

$\sqrt{11 - x^2} = 2, \ \cancel{\sqrt{11 - x^2} = -1}$

$11 - x^2 = 4$

$x^2 = 7$

$x = \pm\sqrt{7}$

7. Solve the following equation. Check if your solution is valid or extraneous.

$(x - 1)^{-\frac{1}{2}}(x - 8) + (x - 1)^{\frac{1}{2}} = 0$

$\frac{(x - 8)}{(x - 1)^{\frac{1}{2}}} + (x - 1)^{\frac{1}{2}} = 0$

$(x - 8) + (x - 1)^1 = 0$

$2x - 9 = 0$

$x = 9/2$

II.   Linear Inequalities

8. Solve the following inequality.

$3x + 10 \le 6x + 12$

$-3x \le 2$

$x \ge -\frac{2}{3}$

9. Solve the following inequality.

$-2(n - 7) > 17$

$n - 7 < \frac{-17}{2}$

$n < \frac{14}{2} + \frac{17}{2}$

$n < \frac{33}{2}$

Rules for Solving Linear Inequalities

Linear inequalities work just like linear equations but with $<$ or $>$ instead of an $=$.

If we **add or subtract** the same number to both sides, the inequality is *maintained.*
If $5 > 3$, then $5 + 2 > 3 + 2$

If we **multiply or divide** by a **positive** number to both sides, the inequality is *maintained.*
If $5 > 3$, then $5 \times 2 > 3 \times 2$

If we **multiply or divide** by a **positive** number to both sides, the inequality is *maintained.*
If $5 > 3$, then $5 \times -2 < 3 \times -2$

10. Solve for x and graph the solution.
$3x - 4 \le 2$
$3x \le 6$
$x \le 2$

11. Solve for x and graph the solution.
$3 - 2x < 7$
$-2x < 4$
$x > -2$

12. Solve for x and graph the solution.
$3 - 5x \ge 2x + 7$
$3 - 7x \ge 7$
$-7x \ge 4$
$x \le \dfrac{-4}{7}$

13. Solve for x and graph the solution.
$4 + 6x < 3x + 2$
$2 < -3x$
$-3x > 2$
$x < \dfrac{-2}{3}$

14. Solve for x and graph the solution.
$3x + 9 \le 6x + 17$
$-3x \le 8$
$x \ge \dfrac{-8}{3}$

15. Solve the following inequality. Write the answer in bracket interval notation.
$22 \le \dfrac{2}{9}(x - 32) \le 42$
$99 \le x - 32 \le 189$
$131 \le x \le 221$
$[131,221]$

16. Solve the following inequality. Write the answer in bracket interval notation.
$-6x - 2 \le -2(-4x - 2) + 6$
$-6x - 2 \le 8x + 4 + 6$
$-6x - 2 \le 8x + 10$
$-14x \le +12$
$x \ge \dfrac{-6}{7}$
$[-\dfrac{6}{7}, \infty]$

Reviewing Interval Notation (Lesson 0.4)

Draw a number line graph to display the following domains:

(a) $\{x| -2 \le x < 3\}$

(b) $\{x | x < 2 \text{ or } x \ge 7\}$

(c) $\{x | x < 0 \text{ or } 1 \le x < 4\}$

Introducing new notation:

Let $[a, b]$ denote an **open interval** between a and b: $\quad \{x | a \le x \le b\}$

Let (a, b) denote a **closed interval** between a and b: $\quad \{x | a < x < b\}$

Let $(a, b]$ or $[a, b)$ denote a **clopen interval** between a and b: $\quad \{x | a \le x < b\}$ or $\{x | a < x \le b\}$.

III.   Absolute Value Equations & Inequalities

17. Solve the following equation for x.

$|x - 19| = 20$

$x - 19 = 20$ and $-(x - 19) = 20$
$x = 39$ and $(x - 19) = -20$
$x = 39$ and $x = -1$

18. Solve the following equation for x.

$|3x + 2| = 2$

$3x + 2 = 2$ and $-(3x + 2) = 2$
$3x + 2 = 2$ and $3x + 2 = -2$
$3x = 0$ and $3x = -4$
$x = 0$ and $x = -4/3$

19. Solve the following equation for x.

$|-7x + 10| + 9 = 10$

$-7x + 10 + 9 = 10$ and $-(-7x + 10) + 9 = 10$
$-7x + 10 = 1$ and $-(-7x + 10) = 1$
$-7x + 10 = 1$ and $-7x + 10 = -1$
$-7x = -9$ and $-7x = -11$
$x = 9/7$ and $x = 11/7$

20. Solve the following equation.

$|x + 8| = |9x - 9|$

$x + 8 = 9x - 9$ and $-(x + 8) = 9x - 9$
$x + 8 = 9x - 9$ and $-x - 8 = 9x - 9$
$-8x = -17$ and $-10x = -1$
$x = \dfrac{17}{8}$ and $x = -\dfrac{1}{10}$

21. Solve the following equation.

$|2x + 4| = 5$

$2x + 4 = 5$ and $-(2x + 4) = 5$
$2x + 4 = 5$ and $2x + 4 = -5$
$2x = 1$ and $2x = -9$
$x = \dfrac{1}{2}$ and $x = -\dfrac{9}{2}$

22. Solve the following equation.

$|40 - x| = 12$

$40 - x = 12$ and $-(40 - x) = 12$
$40 - x = 12$ and $40 - x = -12$
$-x = -28$ and $-x = -52$
$x = 28$ and $x = 52$

23. Solve the following inequality.

$|x + 2| \geq 8$

$x + 2 \geq 8$ and $-(x + 2) \geq 8$
$x \geq 6$ and $x + 2 \leq -8$
$x \geq 6$ and $x \leq -10$

The **modulus** or **absolute value** of a function is the distance of a value from zero.

For example:

$|-9| = 9$ $|3| = 3$ $|-156| = 156$

Consider the equation:

$|x| = 54$, the possible values of x would be
__-54___ and ___54____.

24. Solve the following inequality.

$|x + 5| > 4$

$x + 5 > 4$ and $-(x + 5) > 4$
$x + 5 > 4$ and $x + 5 < -4$
$x > -1$ and $x < -9$

25. Solve the following inequality.

$4|x + 4| - 14 < 2$

$|x + 4| - 14 < \frac{1}{2}$

$|x + 4| < 14 + \frac{1}{2}$

$|x + 4| < \frac{29}{2}$

$x + 4 < \frac{29}{2}$ and $-(x + 4) < \frac{29}{2}$

$x < \frac{29}{2} - 4$ and $(x + 4) > \frac{-29}{2}$

$x < \frac{29}{2} - 4$ and $x > \frac{-29}{2} - 4$

$x < \frac{29}{2} - \frac{8}{2}$ and $x > \frac{-29}{2} - \frac{8}{2}$

$x < \frac{21}{2}$ and $x > \frac{-37}{2}$

26. Solve the following inequality.

$|x + 4| > -8$

$x + 4 > -8$ and $-(x + 4) > -8$
$x + 4 > -8$ and $x + 4 > 8$
$x > -12$ and $x > 4$

27. Solve the following inequality.

$|x - 1| < 2$

$x - 1 < 2$ and $-(x - 1) < 2$
$x < 3$ and $x - 1 > -2$
$x < 3$ and $x > -1$

28. Solve the following inequality.

$|3x - 1| < -9$

$3x - 1 < -9$ and $-(3x - 1) < -9$
$3x - 1 < -9$ and $(3x - 1) > 9$
$3x < -8$ and $(3x) > 10$
$x < -\frac{8}{3}$ and $x > \frac{10}{3}$

29. Solve the following inequality.

$|x - 1| + 7 > 10$

$x - 1 < 2$ and $-(x - 1) < 2$
$x < 3$ and $x - 1 > -2$
$x < 3$ and $x > -1$

30. Solve the following inequality.

$\left|\frac{3x + 9}{3}\right| \leq 3$

$x + 3 \leq 3$ and $-(x + 3) \leq 3$
$x + 3 \leq 3$ and $x + 3 \leq -3$
$x \leq 0$ and $x \leq -6$